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School of Engineering and Science

RESEARCH DIVISION

University Heights, Bronx 53, N. Y.

Department of Meteorology and Oceanography

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ON THE GROWTH OF THE SPECTRUM OF A WIND GENERATED SEA ACCORDING TO A MODIFIED MILES-PHILLIPS MECHANISM

by

Tokujiro Inoue

Technical Report Prepared for
Contr. N62306 - 1589,
Task Order No. 3.

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Acknowledgments

I sincerely thank Professor Willard J. Pierson, Jr. for guidance and encouragement. Without his help, it would have been very difficult for me to make this study. I also thank Miss Ming-Shun Chang who computed the energy balance between the downward energy flux in the atmosphere and the energy dissipated in the breaking waves of a fully developed sea.

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Abstract

A linear differential equation for the change of spectral density can be expressed by

$$\frac{d}{dt} S(\omega; u, x, y, t) = A(\omega, u(t, x, y)) + B(\omega, u(t, x, y)) \cdot S(\omega; u, x, y, t)$$

where $S(\omega; u, x, y, t)$ is the spectrum, and the terms $A(\omega, u(t, x, y))$ and $B(\omega, u(t, x, y))$, can be considered to be due to the Phillips type of resonance and the Miles surface instability growth theory, respectively. To determine these functions, the spectra computed from wave records obtained by the British weather ships, "Weather Reporter" and "Weather Explorer" were used. Also, to make up for the lack of a full range of those data, results obtained in the field study of Snyder and Cox were used.

The spectral growth equation was then extended by an appropriate assumption for energy dissipation. The derived spectral growth agrees with the growth of waves obtained by Sverdrup and Munk in the wave height sense, and the spectra grow in roughly the way postulated by Neumann. The results of this study show that about 30 hours are required to reach 90% of the fully developed significant wave height for a 40 knot wind measured at 19.5 m. Also about 600 nautical miles are needed for 90% full development for a 40 knot wind velocity at 19.5 m. The family of partially developed sea spectra is shown for wind velocities from 20 knots to 45 knots in 5-knot increments for both the duration limited and fetch limited conditions.

In the process of obtaining the spectral growth equation, the Miles mechanism growth rate had to be increased by a factor of eight compared to the values calculated exactly from the theory, and one-twentieth of the Phillips growth rate, as originally estimated by Phillips, was used.

1. Introduction

The need for the study of ocean waves has increased rapidly owing to increased needs in fields like naval architecture, optimum ship routing and coastal engineering. One of the most important problems is the forecasting of sea surface conditions. Many important contributions were made by scientists after World War II that have clarified the nature of the sea. There are still many difficulties since ocean wave problems are not simple, mainly because of the complexity of the turbulence at the interface of the atmosphere and the ocean.

Since the introduction of the wave spectrum concept, ocean wave studies have made remarkable progress. New practical wave forecasting methods using the spectrum were welcomed. The techniques agreed with observations to some degree, but many defects were pointed out. Later, more observations were obtained, and the spectral form of the fully developed sea was studied more carefully.

Wave forecasting had been done by using an empirical spectral growth relationship. In the late 1950's, two kinds of wave generation mechanisms were proposed. In this paper, an attempt to determine spectral wave growth is reported by the application of two modified wave generation mechanisms based on the theories of Miles and Phillips.

2. History

For twenty years or more, the study of ocean waves has progressed remarkably. This progress was initiated by Sverdrup and Munk (1947) in the study of forecasting waves by introducing a new

concept of wave height, the significant wave height. The relation between the wind and the ocean wave was investigated by means of non-dimensional parameters based on steepness and wave age. They presented a wave forecasting method based on these concepts.

Later, the concept of wave spectra (Pierson, 1952) was able to describe the complex phenomenon of ocean waves. Since then, ocean waves have been treated as a composition of many components, and not as a sinusoidal or regularly shaped wave.

Longuet-Higgins (1952) presented statistical relationships for the average height of the one-tenth highest wave, the significant wave height, and the mean wave height in terms of the variance of the wave record.

Then Neumann (1953) presented a spectral form for the waves and the co-cumulative spectral curve as derived from many visual observations and a record obtained off Long Branch, New Jersey. This spectrum was a great step forward in ocean wave forecasting, and it was used in a practical wave forecasting manual by Pierson, Neumann and James (1955).

Bretschneider (1952) revised the Sverdrup-Munk forecasting technique. Thus, the two main schools of wave forecasting techniques were proposed: (i) the significant wave height method by Sverdrup, Munk and Bretschneider, and (ii) wave spectrum method by Pierson, Neumann and James (1955).

As the observations of waves increased, spectral forms were proposed by Darbyshire (1955, 1957), Bretschneider (1959), and many others. Darbyshire also concluded that the significant wave

height was proportional to the square of the wind velocity against the 2.5^{th} power results of Neumann (1953). Later, Moskowitz (1964) obtained the square law. The discrepancies between the various wave heights and the various proposed spectra were studied by Pierson (1964) in terms of the wind profile instead of the wind measured at a constant height, and the wind profile effect brings the various proposed results into closer agreement. These discrepancies were also discussed by Walden in Ocean Wave Spectra (1963).

Phillips (1958) has found the general character of the equilibrium range of the spectrum. A form of $\alpha g^2/\omega^5$ was derived for the equilibrium range, where α is a constant, g is the gravitational acceleration, and ω is the angular frequency. Pierson and Moskowitz (1964) then proposed the spectral form given by

$$S(\omega) = \frac{\alpha g^2}{\omega^5} e^{-\beta(g/\omega u)^4} \tag{2.1}$$

where $\alpha = 8.10 \times 10^{-3}$, $\beta = 0.74$ and u is the wind velocity at 19.5 m height. This spectrum was based on the similarity theory given by Kitaigorodskii (1961) and on averages of selected spectra for wind velocities near 20, 25, 30, 35, and 40 knots.

Although there are various proposed wave spectral forms for partially developed seas, they have not yet been explained in terms of wave generation mechanisms. For the practical purpose of wave forecasting (Baer, 1962, 1965) an empirical growth law can be applied. In this paper, the growth rate will be studied in terms of the wave generation mechanisms.

In general, three mechanisms have been proposed for wave generation. The first one is the asymmetry of the normal pressure pattern over the wavy surface. The second is an instability at the interface of the atmosphere and the water, and the third is a resonance mechanism between the atmospheric pressure fluctuation and the sea.

The first mechanism was proposed by Jeffreys (1925). When the wind blows over the waves which are supposed to exist already, the wind produces a variable pressure distribution, which can be visualized by thinking about the eddies formed on the lee side of the wave crests. The wind adds pressure to the windward side, and sucks on the lee side. This pressure distribution results in the theory of the sheltering coefficient (given as 0.27 by Jeffreys). However, experimental results measured above a solid wave model showed that this value was too high by about one order of magnitude. Sverdrup and Munk (1947) suggested that it was about 0.049. Another defect is that the wave can grow only in the case of a wind velocity higher than the wave phase velocity.

Due to this weak point, Sverdrup and Munk decided that they should take into account not only normal stress but also the tangential stress acting on the sea surface, and they showed that the waves could grow even if the waves moved faster than the wind. By assuming that all the energy transferred to the water by the tangential stress produced waves and not currents, the calculations agreed with the magnitude of the observed wave heights.

However, another problem arose because the tangential stress causes the wave to be rotational. In ocean waves, irrotationality

is mostly found, especially in the early stage of development.

From the point of view of irrotationality, normal pressures influence wave generation in the early generating stage. As mentioned before, Jeffreys considered only normal pressure which seems adequate with reference to irrotationality. However, the empirical assumption of a sheltering coefficient was introduced. Moreover, an important assumption was made that the wave was already in existence. If there were no wave, the pressure distribution would be uniform over the sea surface and induce no sheltering coefficient. The waves would not grow if the sea was at rest initially.

Phillips (1957) studied the generation of waves by a turbulent wind. According to his theory, waves can grow from an initial calm, and waves develop rapidly by the resonance mechanism which occurs when a component of the surface pressure distribution moves at the same speed as the free surface wave with the same wave number.

With another approach, Miles (1957) overcame the uncertain assumptions which were involved in the concepts of the sheltering coefficient and the tangential energy transfer. The amplitude and phase of the pressure fluctuations can be computed in terms of the characteristics of the wind profile with no unknown constants. The energy transfer from the wind to the waves is calculated, with the result that the mean rate at which energy is transferred from a parallel shear flow to a surface wave is proportional to the curvature of the wind profile at the height for which the mean wind velocity is the same as the phase velocity and inversely proportional to the slope of the wind profile.

Thus we now know the spectral form for a fully developed sea, and two theoretical wave generation mechanisms. A study of wave growth in terms of these theories should improve practical forecasting.

3. Phillips resonance theory and Miles instability theory

The resonance mechanism will be discussed first. The atmospheric eddies are associated with random stress fluctuations on the surface, both normal pressures and tangential shear stresses. The eddies are advected by the mean velocity of the wind, and at the same time they develop, interact and decay, so that the associated stress distribution moves over the water surface with a certain convection velocity dependent upon the velocity of the wind and changes in time.

This stress distribution contains components with a very large range of wave numbers, and each component is convected at a different speed. These components of the pressure fluctuations acting on the sea surface generate small forced oscillations, and the amplitude of any component of the surface displacement depends upon the amplitude of the corresponding component of the forcing pressure fluctuations. However, the response of the water surface to the various components of the pressure field is not uniform, because certain combinations of wave number and frequency of the components of the sea are excited more rapidly than others. This resonance feeds more energy to that particular component.

Assume that the pressure fluctuations and the displacement of the sea surface are stationary random functions of position. By introducing a Fourier-Stieltjes integral, the displacement of the sea surface in terms of a random prosess $A(\vec{K}, t)$.

$$\xi(\vec{x}, t) = \int e^{i\vec{K}\cdot\vec{x}}\, dA(\vec{K}, t) \tag{3.1}$$

and the two-dimensional instantaneous spectrum of the surface displacement is expressed by the Fourier transform of the covariance function

$$R(\vec{r}) = \overline{\xi(\vec{x})\, \xi(\vec{x} + \vec{r})} \tag{3.2}$$

Then the spectrum of the sea can be shown as

$$S(\vec{K}, t) = \frac{\overline{dA(\vec{K}, t)\, dA^*(\vec{K}, t)}}{dK_1\, dK_2} \tag{3.3}$$

Similarly, the spectrum of the surface pressure fluctuations can be written in the form

$$\Pi(\vec{K}, t) = \frac{\overline{d\bar{\omega}(\vec{K}, t')\, d\omega^*(\vec{K}, t' + t)}}{dK_1\, dK_2} \tag{3.4}$$

The potential equation can then be obtained under the assumption of irrotational motion in the water.

When the sea is at rest initially, the solution is shown as

$$dA(\vec{K}, t) = \frac{iK}{2\rho n_2} \int_0^t d\bar{\omega}(t)[\exp\{-i(n_1 - n_2)(\tau - t)\}$$

$$- \exp\{-i(n_1 + n_2)(\tau - t)\}]\, d\tau \tag{3.5}$$

where $$n_1 = \vec{K} \cdot \vec{U}$$

$$n_2 = \left(gK + \frac{TK^3}{\rho}\right)^{1/2}$$

and T is the surface tension.

This shows how the amplitude of each Fourier-Stieltjes component of the wave depends upon the history of the given pressure distribution from zero initial time.

For the principal stage of development and for gravity waves, the wave spectral function is

$$\Phi(\vec{K}, t) \sim \frac{\Pi(\vec{K}, t)}{2\sqrt{2}\,\rho^2 g(U - c(K))} \tag{3.6}$$

where U is the convection speed of the atmospheric eddies, and $c(K)$ is the phase velocity of the wave with a component of wave number K.

If $U >> c(K)$ by a factor of 3 or 4, equation (3.6) becomes

$$\Phi(\vec{K}, t) \sim \frac{\Pi(\vec{K}, t)}{2\sqrt{2}\,\rho^2 Ug} \tag{3.7}$$

For the mean square wave growth, this yields

$$\overline{\xi^2} \sim \frac{\overline{p^2}\, t}{2\sqrt{2}\,\rho^2 Ug} \tag{3.8}$$

where $\overline{p^2}$ is the mean square pressure fluctuations.

The mean square pressure fluctuation is assumed from oceanographic measurements to be given by

$$\overline{p^2} \simeq 9 \times 10^3\, \rho_a^2\, u_*^4 \tag{3.9}$$

at 5 m height, where u_* is a friction velocity. The friction velocity is approximated as $U = 18\, u_*$.

Then (3.9) becomes

$$\overline{p^2} \simeq 9 \times 10^{-2}\, \rho_a^2\, U^4 \tag{3.10}$$

and finally the expression for the mean square wave growth is

$$\overline{\xi^2} \sim 0.035 \left(\frac{\rho_a}{\rho_w}\right)^2 \frac{U^3}{g}\, t \tag{3.11}$$

Equation (3.7) was derived from equation (3.6) under the assumption that $U >> c(\kappa)$. The spectral growth is important near the frequency where $U \simeq c(\kappa)$. The assumption made above is not appropriate from this point of view. Equation (3.11) then becomes

$$\overline{\xi^2} \sim \gamma\left(\frac{\rho_a}{\rho_w}\right) \frac{U^4 t}{|U - c(\kappa)| g} \tag{3.12}$$

Equation (3.12) has a singularity in the solution because $\overline{\xi^2}$ approaches plus infinity as U approaches $c(\kappa)$. The field study made by Snyder and Cox (1965) suggests no evidence of this type of curve for U near $c(\kappa)$. Thus the result (3.11) as modified is applied later as the initial growth for all values of U.

Miles (1957) studied the wave generation mechanism in terms of an instability theory. The vertical wind profile was assumed to be some functions $U(z)$, with no mean water motion due to the flow of air over the sea surface, and the flows were considered to be inviscid. Under these conditions, the energy is transferred by normal pressures.

The rate of energy extraction from the mean flow is

$$\frac{dE}{dt} = \rho_a \int_0^\infty \overline{uw} \, \frac{\partial u}{\partial z} \, dz \tag{3.13}$$

For an inviscid parallel flow, this leads to the following approximation:

$$\rho_a \overline{uw} \simeq \rho_a \frac{\pi}{\kappa} \frac{U''}{U'}\bigg|_{z=z_c} \cdot \overline{w^2}\bigg|_{z=z_c} \tag{3.14}$$

where U'' is the curvature of the wind profile at the height where $U = c$ and U' is the slope of the wind profile at the same height above.

Then (3.13) becomes

$$\overline{\frac{dE}{dt}} \propto -\frac{\rho_a \pi c}{\kappa}\left(\frac{U''}{U'}\right)\Bigg|_{z=z_c} \cdot \overline{w^2}\,\Bigg|_{z=z_c} \tag{3.15}$$

Thus, the mean rate at which energy is transferred from a parallel shear flow $U(z)$ to a surface wave of wavelength $\overline{L} = 2\pi/\kappa$ and wave phase speed c is proportional to the curvature and inversely proportional to the slope of the wind profile at the height where $U = c$.

From equation (3.15), the general tendency of wave component growth is explained. The longer the wave component, or the larger the phase velocity c, the slower the rate of energy transfer. At first, the short waves will dominate.

One other conclusion of Miles is that measurements of the pressure distribution on a stationary wave model with wind blowing over it would not necessarily result in sheltering coefficients that would be similar to those obtained if the wave surface were moving at the speed c.

The essential feature of Miles' theory is that the air pressure is out of phase with the water surface. The water surface is

$$\eta = \mathcal{R}e\,\{a e^{i(kx-\sigma t)}\} \tag{3.16}$$

and the aerodynamical pressure exerted on a boundary defined by

(3.16) is

$$p = \mathcal{R}e\{(\alpha + i\beta)\rho_a \kappa u_1^2 \eta\} \tag{3.17}$$

where α and β are real, non-dimensional quantities depending on the wind profile, ρ_a is the density of the air, u_1 is u_*/k (u_* is friction velocity and k is a Karman constant), and κ is a wave number.

The energy per wavelength transferred from the air to the water is given by

$$E_L = \pi\rho_a g a^2 c \left(\frac{u_1}{c}\right)^2 \sqrt{\alpha^2 + \beta^2}\ \sin\phi \tag{3.18}$$

where ϕ is the angle by which the maximum air pressure leads the wave trough, and

$$\tan\phi = -\beta/\alpha \tag{3.19}$$

Then the maximum air pressure occurs before the wave trough, and the minimum occurs a little before the wave crest.

Miles (1959) evaluated the result of the mean energy transfer rate in terms of an assumed logarithmic wind profile by introducing a function β, and concluded that

$$\overline{\frac{dE}{dt}} = \rho_a u_1^2 \int_0^\infty \int_{-\frac{1}{2}\theta_o}^{\frac{1}{2}\theta_o} c\,\beta\left(\frac{c}{u_1 \cos\theta}\right)\kappa^2 S(\kappa,\theta)\cos^2\theta\,\kappa\,d\kappa\,d\theta \tag{3.20}$$

where θ is the angle between the wind and the direction of propagation of the wave, and $S(\kappa, \theta)$ is the power spectral density of the surface displacement.

4. Spectrum equation

When the spectral value of a component is relatively small so that nonlinear effects can be neglected, the differential equation for the spectral growth can be expressed by

$$\frac{d}{dt} S(\omega; t, x, y) = \frac{\partial}{\partial t} S(\omega; t, x, y) + W(\omega) \cdot \nabla S(\omega; t, x, y)$$
$$= A(\omega; u(t, x, y)) + B(\omega; u(t, x, y)) \cdot S(\omega; t, x, y) \quad (4.1)$$

where $S(\omega; t, x, y)$ is the spectral density of a frequency component, and A and B are unknown functions of frequency and wind speed as a function of space and time. In what follows, it will be assumed that u is a constant and that the spectrum is either time-varying or space-varying, but not both.

If the sea is at rest initially, then the spectrum is zero for all wave numbers, or frequencies. The sea must grow by the Phillips' resonance mechanism in which the first term of the last expression, $A(\omega; u)$ corresponds with this resonance mechanism. Once the sea is formed, the Miles' instability growth mechanism can have an effect as the second term, $B(\omega; u) \cdot S(\omega; t)$.

From equation (3.11), the mean square value of the sea surface displacement is a linear function of time. So the term $A(\omega, u)$ can be considered to be

$$A(\omega, u) \sim 0.035 \left(\frac{\rho_a}{\rho_w}\right)^2 \frac{U^3}{g} \quad (4.2)$$

In the next section, the term $B(\omega, u)\, S(\omega; t)$ is investigated under the assumption that u is constant with respect to time. The energy transfer rate by the normal pressure from shear flow is

derived in the form of (3.20). However, the unknown function $\beta(c/U_1 \cos\theta)$ is included. Miles (1959) showed some relation between β and c/U_1 with respect to a few different Ω values, where $\Omega = gz_o/U_1^2$ and z_o is a roughness parameter. The function $\beta(c/U_1 \cos\theta)$ should be known.

In this paper, only one dimensional spectral growth can be considered because observational data to be used are one-dimensional. Thus equation (3.20) is integrated from -90° to +90° about the wind direction.

Equation (3.20) thus becomes

$$\overline{\frac{dE}{dt}} = \rho_a U_1^2 \int_o^\infty c\,\beta\left(\frac{c}{U_1}\right)\kappa^2 S(\kappa)\,\kappa\,d\kappa \tag{4.3}$$

From a dimensional analysis, (4.3) becomes

$$\overline{\frac{dE}{dt}} = \left(\frac{\rho_a}{\rho_w}\right)\int_o^\infty \left(\frac{\omega U_1}{g}\right)^2 \omega\beta\left(\frac{c}{U_1}\right) S(\omega)\,d\omega$$

or

$$\overline{\frac{dE}{dt}} = C'\beta\left(\frac{c}{U_1}\right)\int_o^\infty \left(\frac{U_1}{c}\right)^2 \omega S(\omega)\,d\omega \tag{4.4}$$

where C' is a constant.

The constant C' and the function $\beta(c/U_1)$ need to be determined independently by observation. The $\beta(c/U_1)$ function is considered to be expressed in the form of an exponential function to keep the same general form in terms of frequency that appeared in Miles (1959).

The energy rate of change can be written as

$$\frac{d}{dt} S(f) = D(U/c)^2 e^{-E(c/U)^n} \cdot f \cdot S(f) \qquad (4.5)$$

where D, E, and n are constants and where the Phillips' term is omitted. These constants are to be obtained by using wave spectra obtained from observations of waves made in the North Atlantic.

5. Data and preliminary analysis

The unknown constants D, E, and n must be determined to get the $B(\omega, u) S(\omega; t)$ term in equation (4.1). Wave observations in the North Atlantic were used for this purpose.

The spectra obtained from wave records obtained by the British weather ships "Weather Reporter" and "Weather Explorer" were studied for the period from April 1955 to March 1960. These waves were measured with the Tucker shipborne wave recorder at the ocean weather stations A, I, J, and K located as listed below:

Station A :	62°N, 33°W
Station B :	59°N, 19°W
Station J :	52.5°N, 20°W
Station K :	45°N, 16°W

From these wave data, 460 wave records were picked and spectrally analyzed by Moskowitz, Pierson and Mehr (1962, 1963, 1965). Each record was approximately fifteen minutes long and the spectra have a frequency range from 0 to 0.333 cycles per second. The wave observations were made at three or six hour intervals.

These 460 wave records include fully developed seas, growing seas, decaying seas and swells. For this study, only growing sea

spectra are required and little change of the meteorological conditions is preferable.

From this point of view, three criteria were laid down: (i) the significant wave height was increasing, (ii) the change of wind velocity was less than two knots in three hours, (iii) the change of wind direction was less than 45 degrees within three hours. After choosing the spectra which passed the above criteria, they were checked again from the spectral point of view. The spectra that contained dominant values in the lower frequency region, or swell, and two or more significant peaks were rejected. Under these conditions, only ten wave spectral pairs, or so, passed. The amount of data that can be used to this study is not enough, but these final spectra seem to be really growing stage spectra.

Equation (4.1) has two unknown functions. The Phillips' term, $A(\omega, u; t)$, was expressed by equation (4.2) and changed into more preferable units for ease of comparison with the observed data. The wind velocity on the British weather ships is measured at 19.5 m height and the spectra have their variance in ft^2 as integrated over a frequency band width of $1/180 \ sec^{-1}$. The value of the A term was changed into those units, and the mean square surface displacement was spread out uniformly over all frequency band widths.

In these units, (4.2) is expressed by

$$A \simeq 2.8 \times 10^{-7} \times U_{19.5}^3 \quad (ft^2 \ hour^{-1}) \qquad (5.1)$$

where $U_{19.5}$ is a wind velocity in knots at 19.5 m. This shows

that even for quite high winds, say 40 knots, this term does not contribute much to the wave spectrum in the stage for which observed data were obtained. Furthermore, Longuet-Higgins et al† obtained data that suggests the mean square pressure fluctuation assumed by Phillips might be too high by a factor of ten to one hundred. Therefore, for this part of the analysis, the A term can be supposed to be very small so as to get the growth rate for the B term. For this reason, the growth from one spectrum to the next was treated as due only to the Miles term.

Figure 1 shows the growth rate against the variable U/c, U is the wind velocity in knots at 19.5m, c is the phase velocity of the frequency, f, S_o is the initial spectrum, and S_1 is the spectrum after Δt hours.

The plotted points show that the growth rates are smaller in the region for $U/c > 0.9$. In that region, most of the spectra were close to fully developed spectra, and growth rates were smaller. In this study, nonlinear effects are probably important for $U/c > 0.9$. Thus not all of the plotted points can be used to estimate the Miles term. More data are needed, especially in the region of high U/c.

A field study of wind wave generation was made by Snyder and Cox (1965) in the Bahamas. The growth of a single wave component was studied for a wind velocity range from 5 knots to 20 knots. These measurements were used and converted into U/c units. They are scattered in the high U/c range as shown by the

† Ocean Wave Spectra (1963), p. 129.

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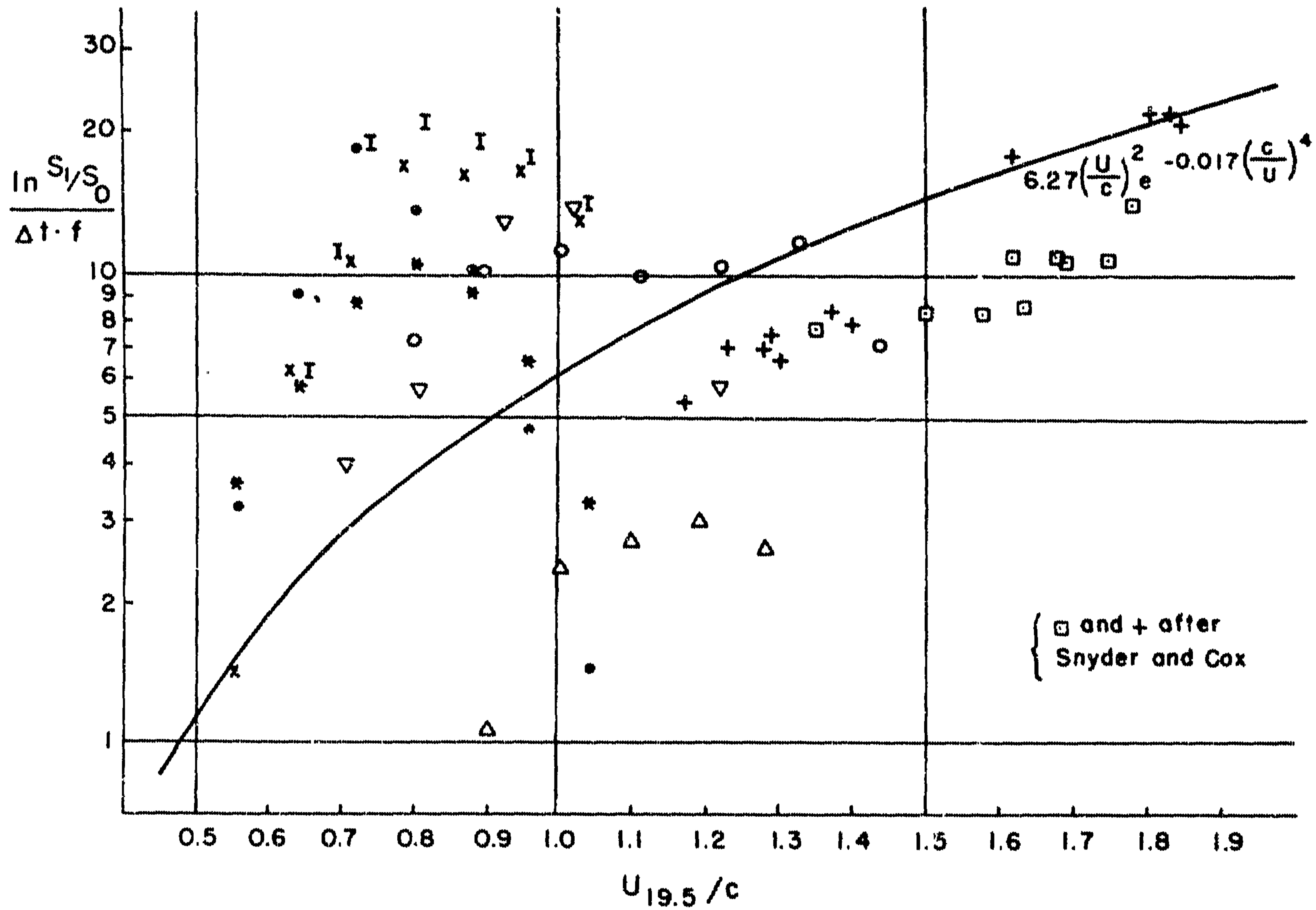

Fig. 1. B term growth scatter diagram as a function of $U_{19.5}/c$. (Code symbols are listed in the Appendix.)

coded points.

By combining these data with the deep ocean data, the following expression was obtained:

$$B(f, u_*) = 6.27\,(U/c)^2\, e^{-0.017(c/U)^4} \cdot f \qquad (\text{hour}^{-1}) \qquad (5.2)$$

where f is in $\sec^{-1}$.

Snyder and Cox (1965) also estimated the A, or Phillips term. Figure 2 shows one-tenth and one-twentieth of the original Phillips estimate, and the fit of the points. This figure seems to verify the estimate made by Longuet-Higgins et al (1963). The U^3 dependence is not clearly established, but the scattered points seem to suggest it.

However, Snyder and Cox suggested that the general trend of the B term, or Miles term, is approximately linear. According to the diagram of their paper, the B term becomes zero in the lower range at approximately, $U/c = 0.7$. The plotted points in that diagram, however, seem to be a square or some power law for U/c rather than a linear relationship. If the linear relation holds, the wave spectral density does not grow below this particular frequency due to the Miles term.

From equations (5.1) and (5.2), the A term grows as a U^3 law and the B term grows as $(U/c)^2$. These results seem to agree quite well with the results obtained by Snyder and Cox.

In this study, a value of one-twentieth of the original estimate is used for the Phillips term. Hence, the equation is written as

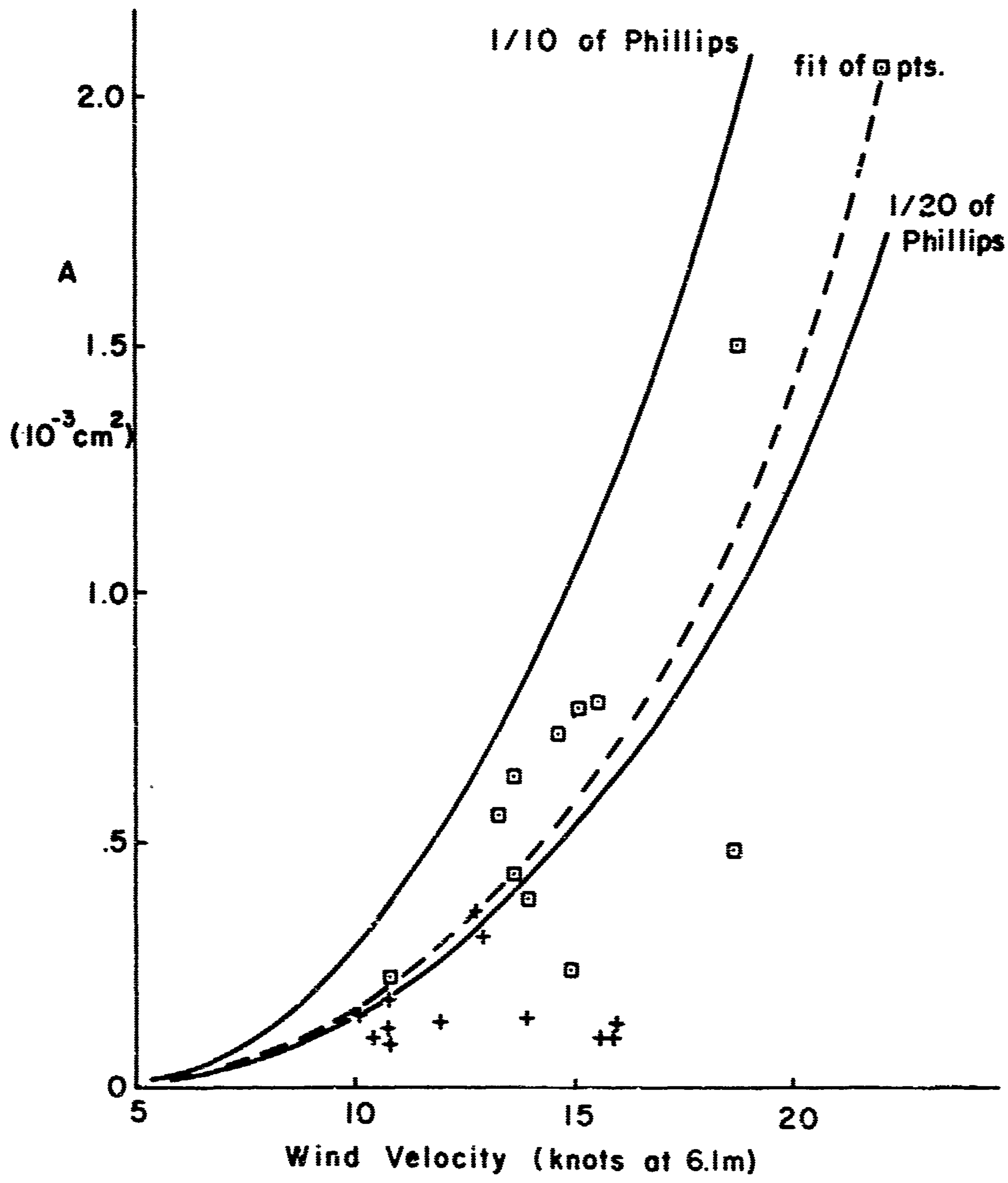

Fig. 2. A term growth diagram as a function of wind velocity.

$$\frac{d}{dt} S(f;t)\Delta f = 1.4 \times 10^{-8} U^3 + 6.27 (U/c)^2 e^{-0.017 (c/U)^4} \cdot f \cdot S(f;t) \Delta f \qquad (5.3)$$

$$(ft^2 \ hour^{-1})$$

for $\Delta f = 1/180 \ sec^{-1}$.

6. Effect of dissipation near full development

According to equation (5.3), the spectrum increases with time to infinite values. However, once the waves start to grow, energy dissipation is supposed to follow more or less immediately. Regretably, dissipation has not been studied very much, and little is known about it.

There is a general agreement upon the sequence of spectral growth. The variance increases slowly after starting from zero initial condition, and the dissipation is not dominant. Thus, it grows slowly and linearly with time in the early stage. Later, when the spectral component comes close to the steady state, energy dissipation increases strongly, and the growth rate is slowed down. As the input energy from the wind and the energy dissipated become equal, the fully developed spectrum is maintained. Between these two stages, the initial and saturated stages, there are the transitional and exponential growth stages.

By considering these growth curves for the spectral wave components, energy dissipation can be assumed to be a function of the ratio of the spectrum at a particular moment to the fully developed spectrum, and as a good approximation may be expressed in the form of

$$\frac{d}{dt} S(f,t)\Delta f = [A(f,u) + B(f,u) S(f,t)\Delta f] \left[1 - \left(\frac{S(f,t)}{S_\infty(f)}\right)^2\right] \qquad (6.1)$$

where $S_\infty(f)$ is the fully developed spectrum for a particular wind speed.

7. Spectral growth equation and calculation

To make the manipulation of (6.1) easier, another function is introduced in the A term which does not influence the growth very much. Equation (6.1) is changed to the form

$$\frac{d}{dt} S(f\ t)\Delta f = \left[A\left\{1 - \left(\frac{S}{S_\infty}\right)^2\right\}^{\frac{1}{2}} + BS\,\Delta f\right] \cdot \left[1 - \left(\frac{S}{S_\infty}\right)^2\right] \qquad (7.1)$$

The solution to this equation for a zero initial spectrum and an infinite fetch is

$$S(f,t)\Delta f = \frac{A\{\exp(Bt) - 1\}}{B} \left[1 + \frac{A^2\{\exp(Bt) - 1\}^2}{(B\,S_\infty \Delta f)^2}\right]^{-\frac{1}{2}} \qquad (7.2)$$

where $A = 1.4 \times 10^{-8} U^3$, $B = 6.27 (U/c)^2 e^{-0.017 (c/U)^4} \cdot f$ and t is the duration in hours.

If the sea is not at rest initially, equation (7.3) may be used.

$$S(f,t)\Delta f = \frac{A[\exp\{B(t+t_o)\} - 1]}{B} \left[1 + \frac{A^2\{\exp(B(t+t_o) - 1)\}^2}{B^2 (S_\infty \Delta f)^2}\right]^{-\frac{1}{2}} \qquad (7.3)$$

where

$$t_o = \frac{1}{B} \ln \left[1 + \frac{B\,S_o \Delta f}{A\left(1 - \left(\frac{S_o}{S_\infty}\right)^2\right)^{\frac{1}{2}}}\right]$$

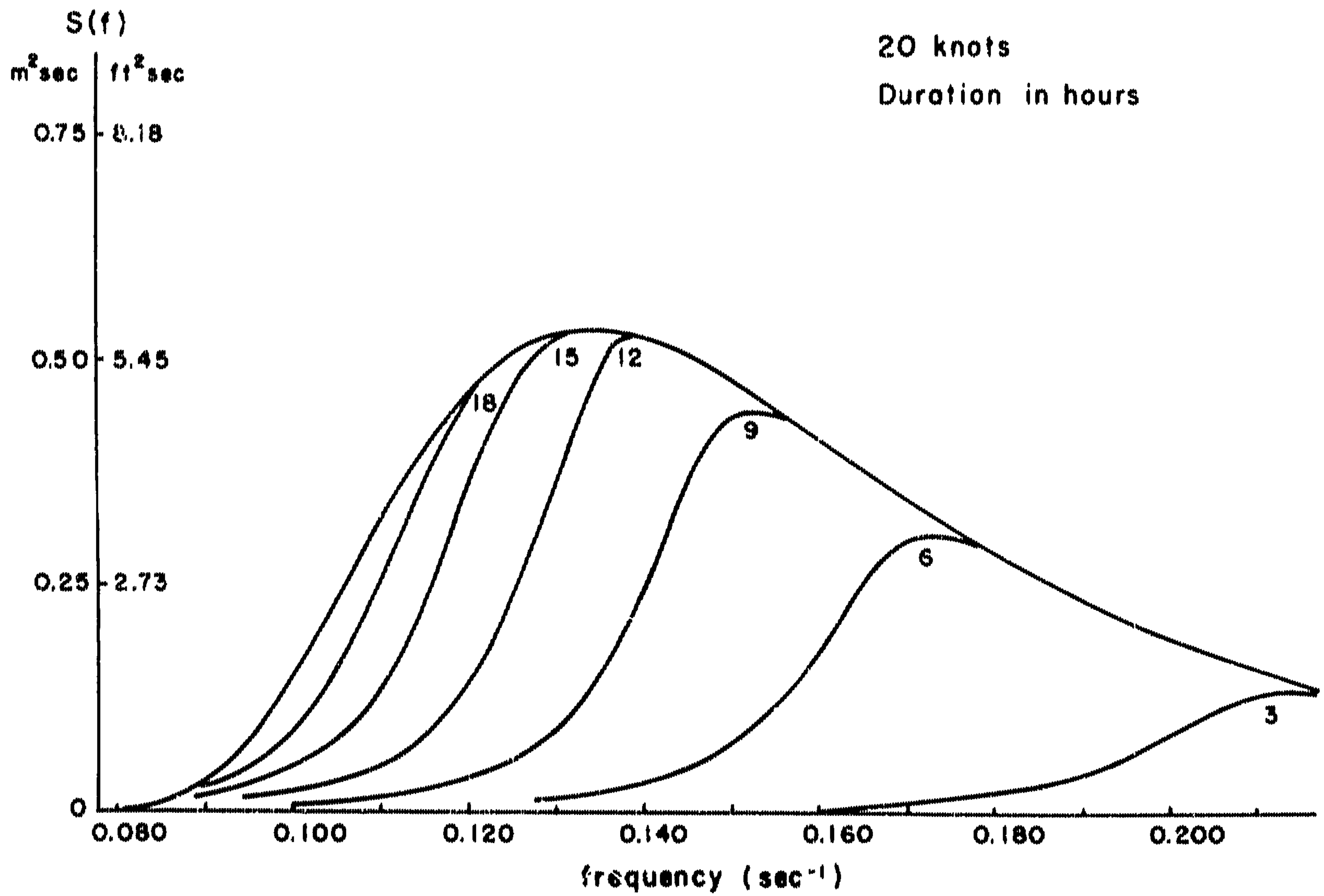

Fig. 3(a) Spectral growth with respect to duration.

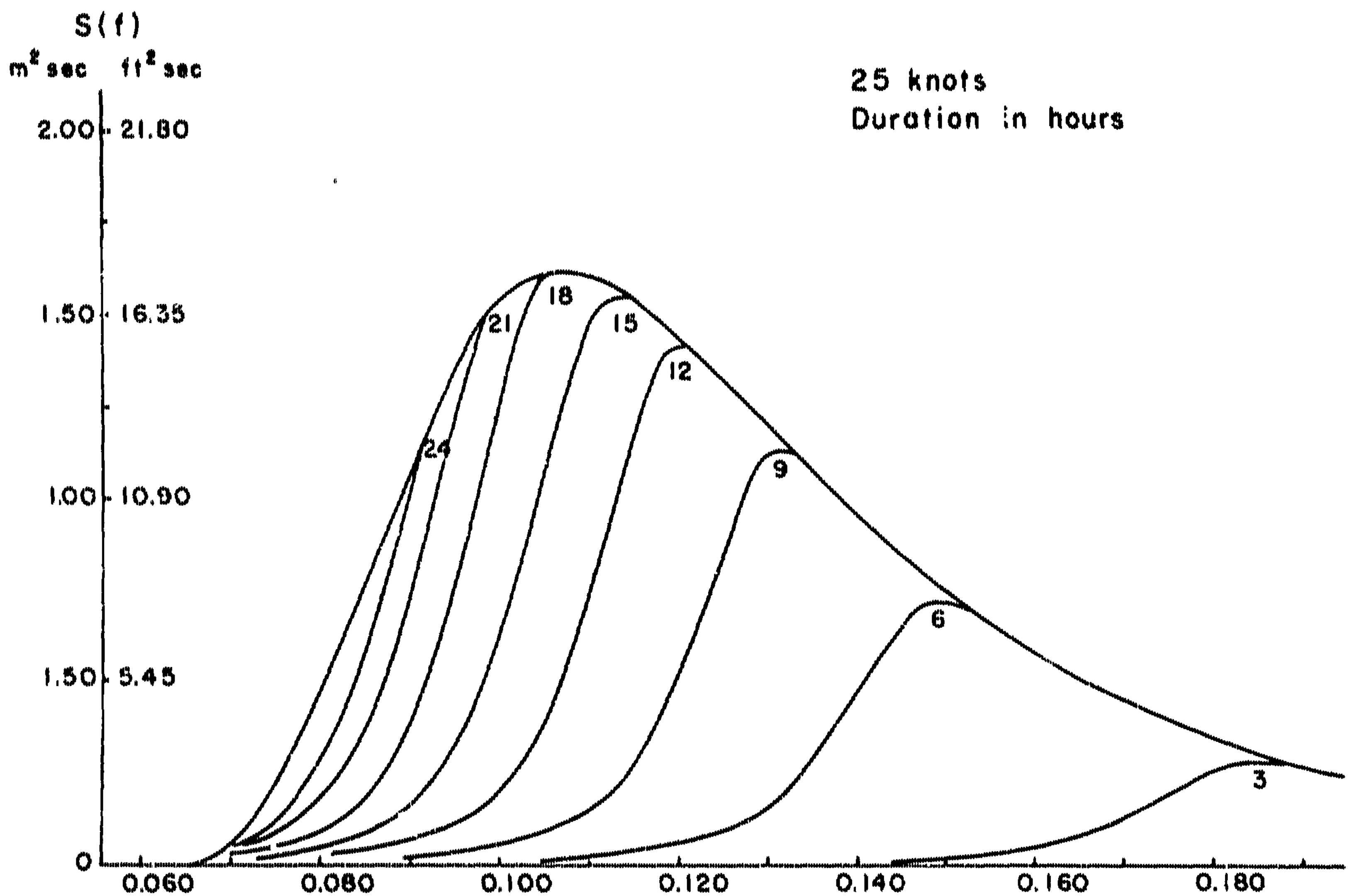

Fig. 3(b) Spectral growth with respect to duration.

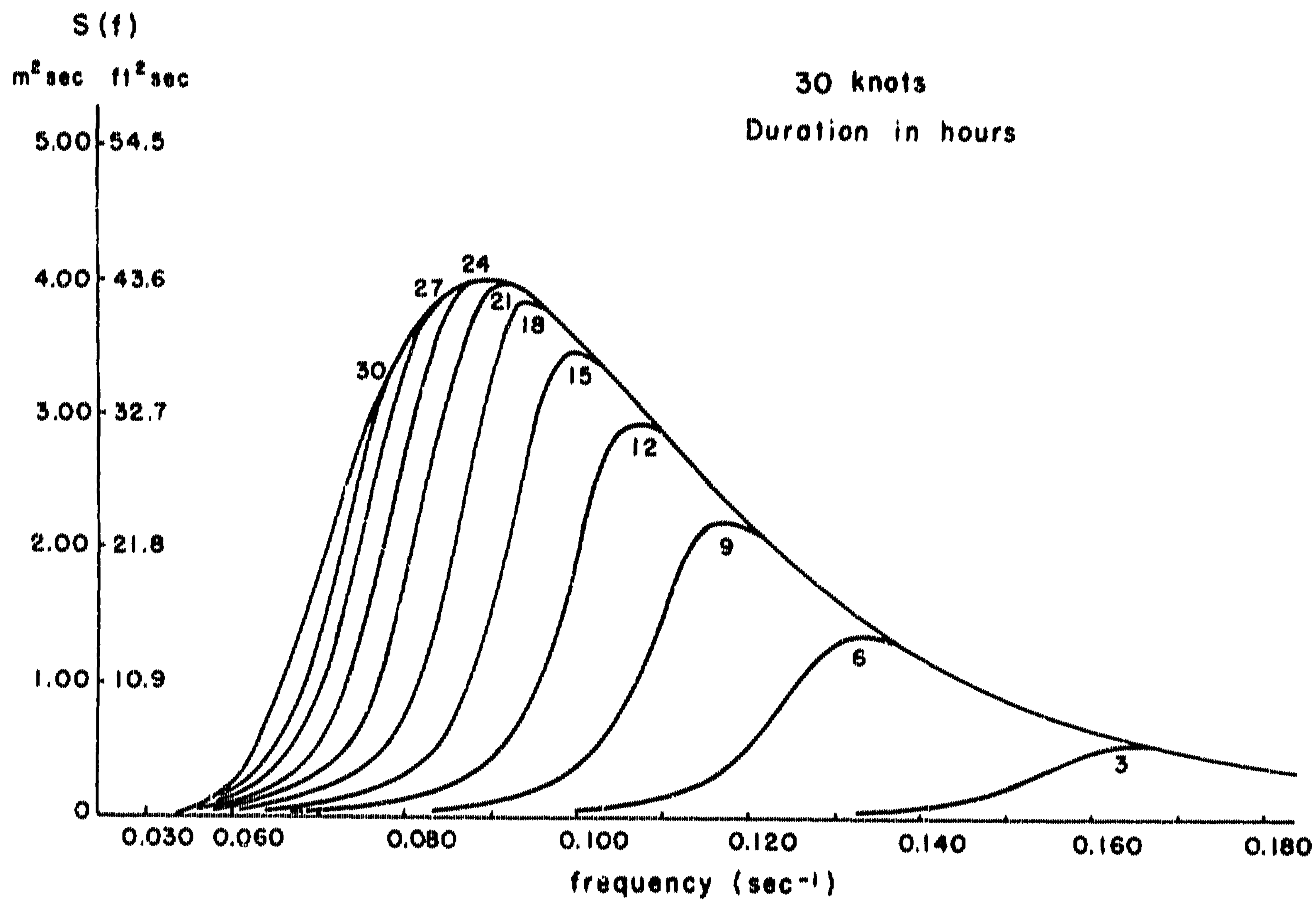

Fig. 3(c) Spectral growth with respect to duration.

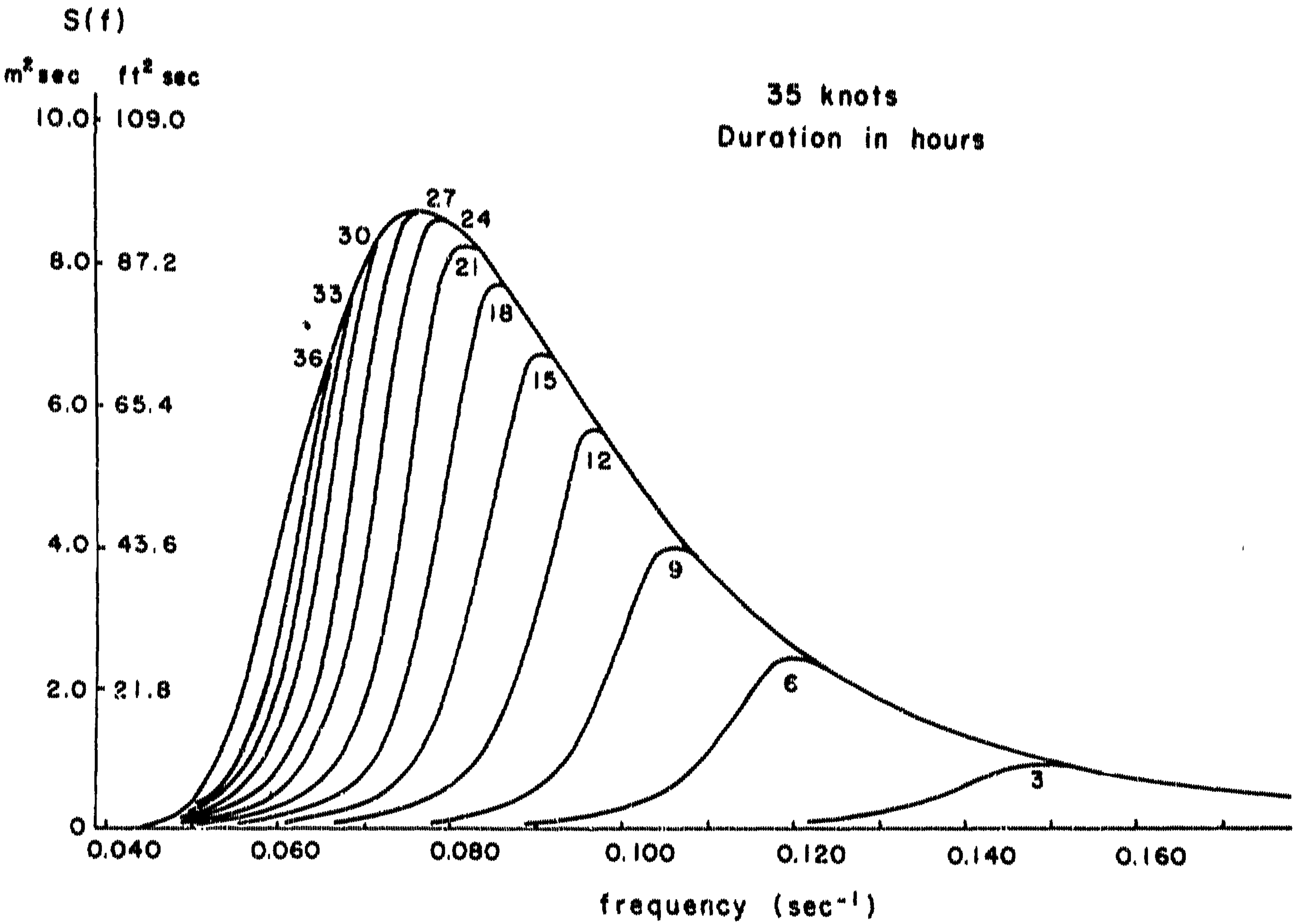

Fig. 3(d) Spectral growth with respect to duration.

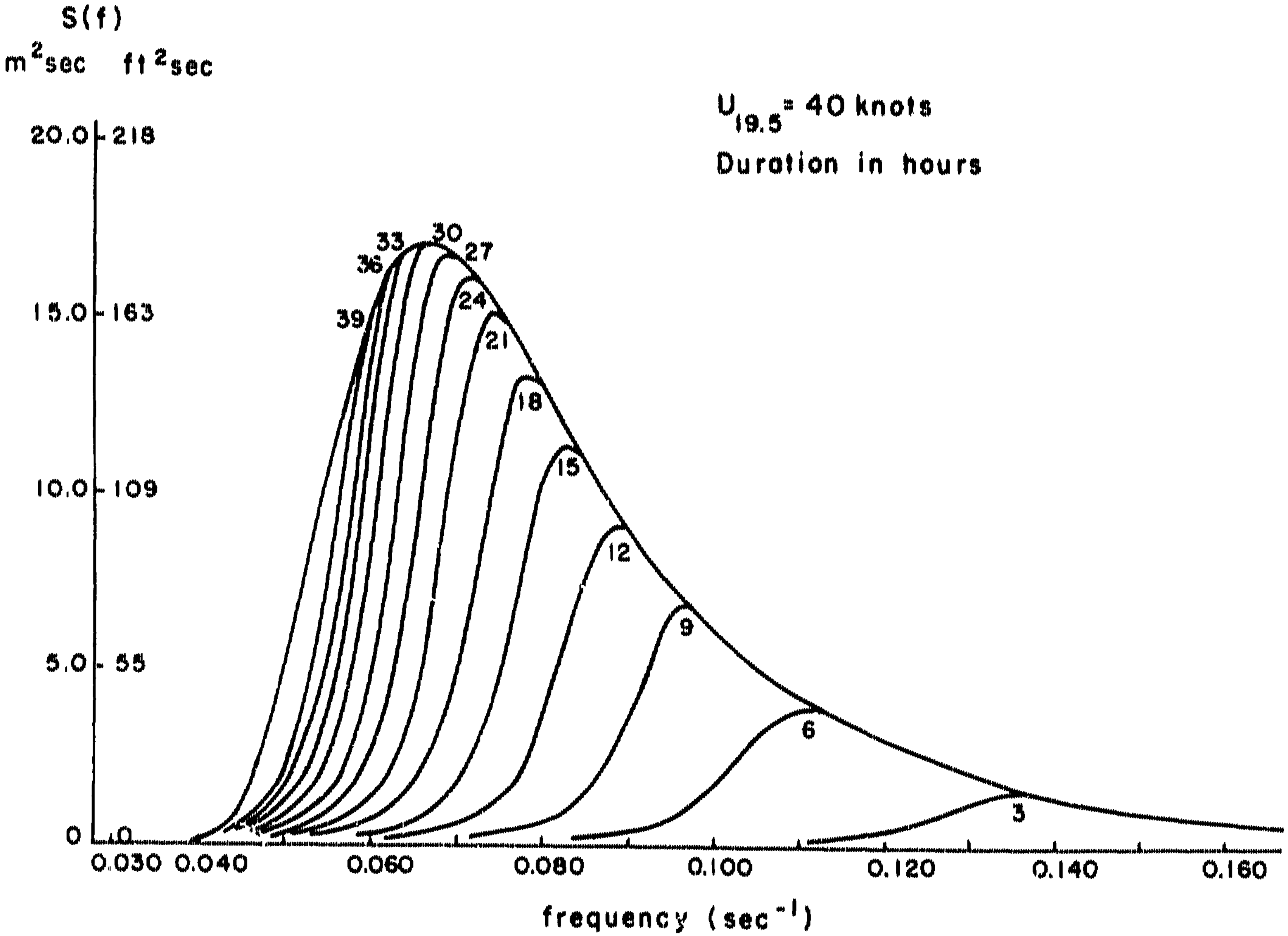

Fig. 3(e) Spectral growth with respect to duration.

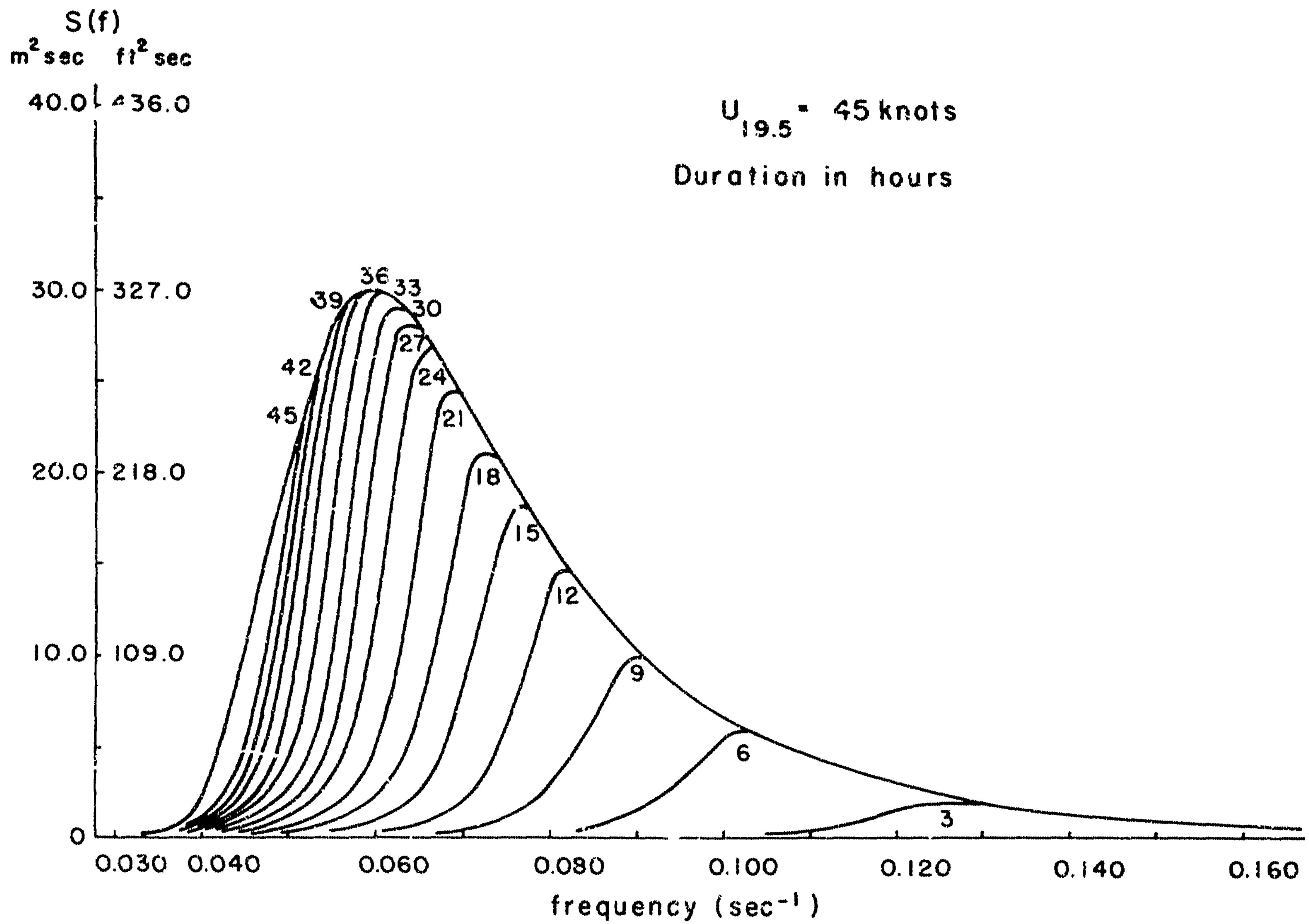

Fig. 3(f) Spectral growth with respect to duration.

and S_o is an initial spectral component.

The spectral growth for wind velocities of 20, 25, 30, 35, 40, and 45 knots was computed by using equation (7.2). In the calculation, the fully developed spectra (equation (2.1)) obtained by Pierson and Moskowitz (1964) was used. These spectral growths are shown in figures 3(a) through (f) as a function of wind duration.

If the wind has blown constantly for a long time, the sea is considered in a steady state. The growth is then independent of time and the differential equation becomes

$$\frac{d}{dt} S(\omega; x) = u_g \frac{\partial}{\partial x} S(\omega; x) = A(\omega, u) + B(\omega, u) S(\omega; x) \qquad (7.4)$$

where u_g is a group velocity of a wave component of frequency f.

A similar computation for fetch was performed under the same wind velocity conditions as for duration. Figures 4 (a) through (f) show the spectra for different fetches for wind velocities of 20 through 45 knots.

These computations were made for modified Miles and Phillips values. However, the spectra in terms of the original Miles and Phillips values should be considered to check the spectra obtained in this paper.

A reduced value for the Miles term of this paper was used to fit his original value. The Phillips term was increased by a factor of twenty and the Miles term was divided by eight. The spectral shape became very different from the spectra obtained in this study. Due to the large Phillips term, high variance occurred in the low frequency range, and a quite flat spectral shape was

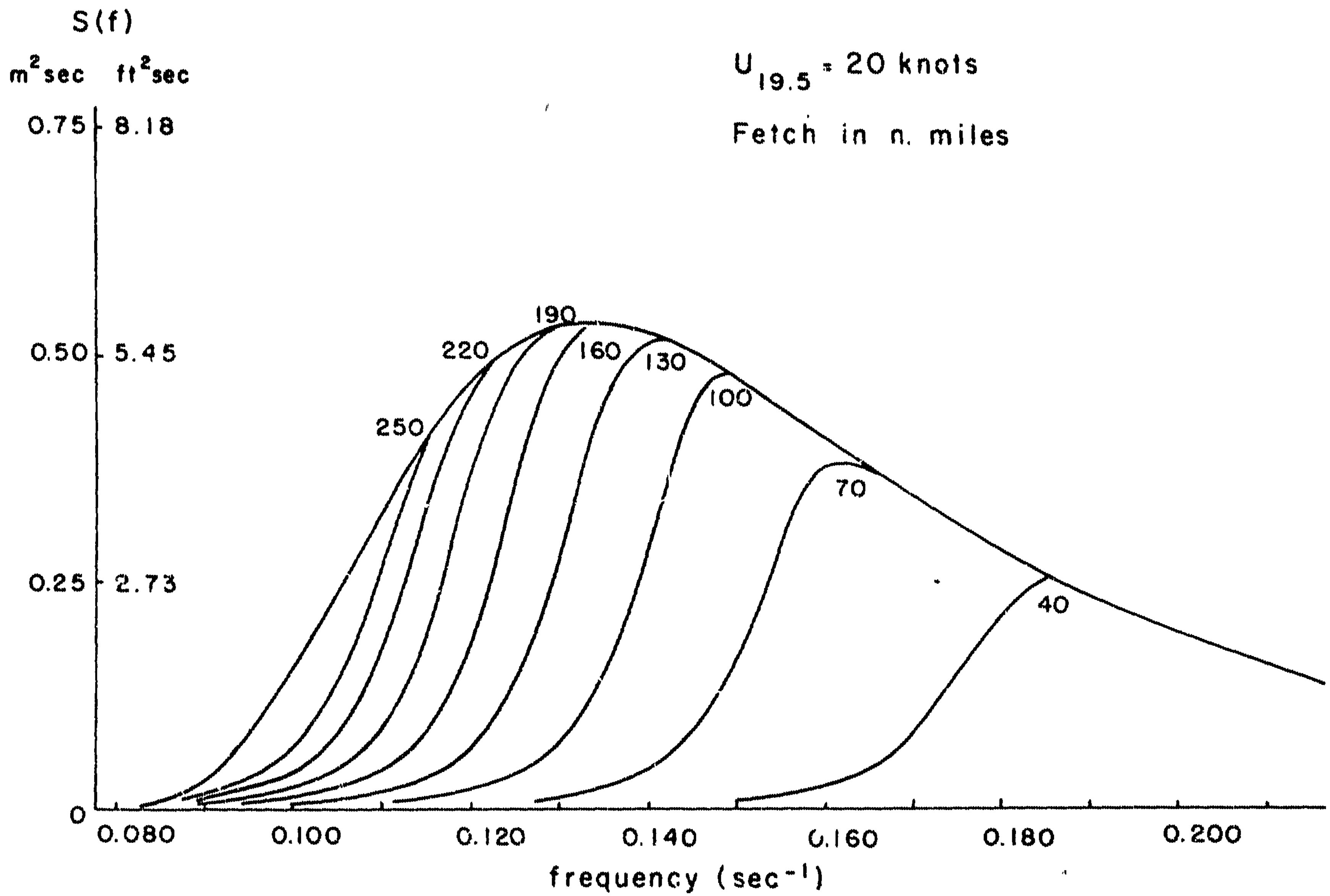

Fig. 4(a) Spectral growth with respect to fetch.

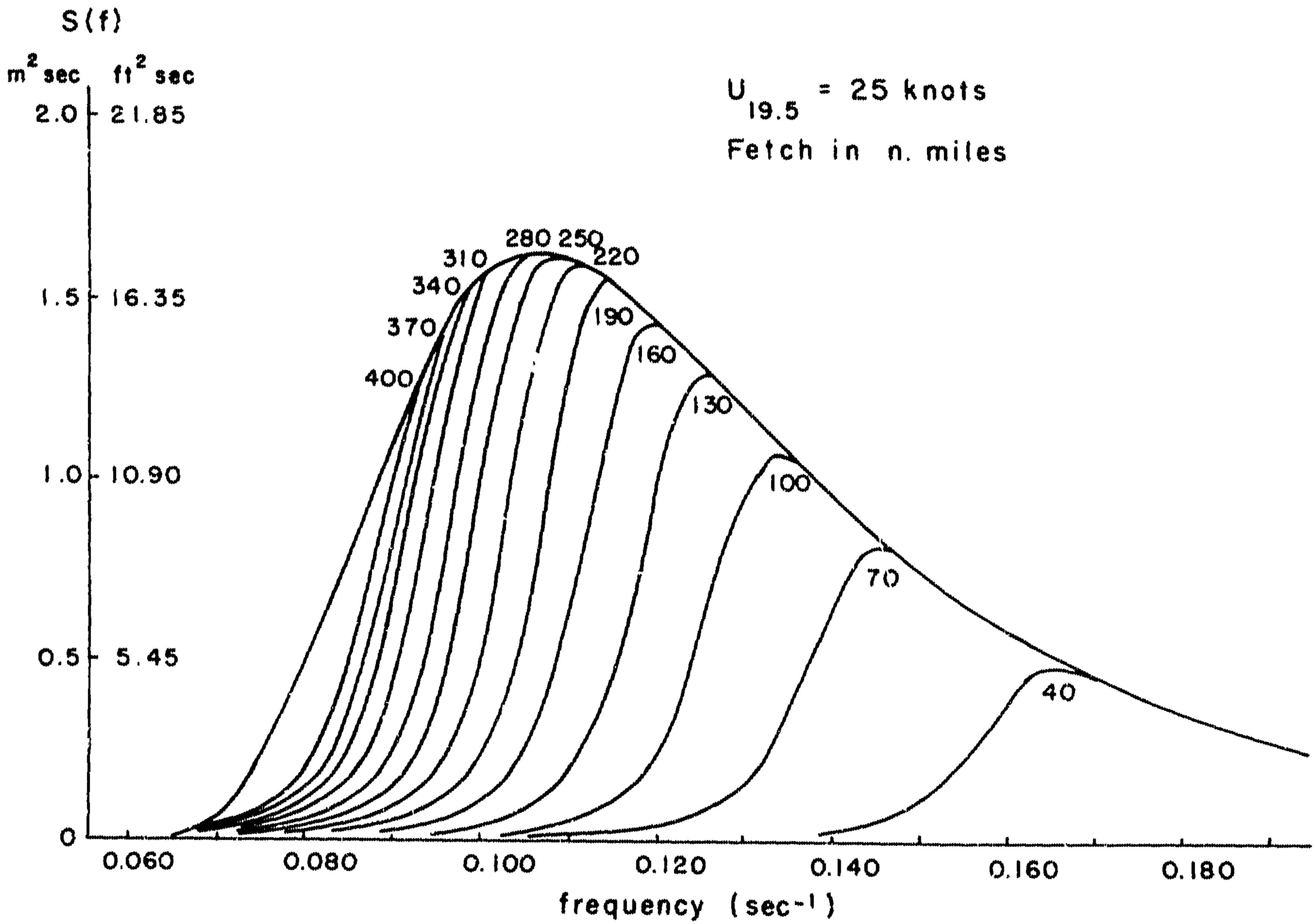

Fig. 4(b) Spectral growth with respect to fetch.

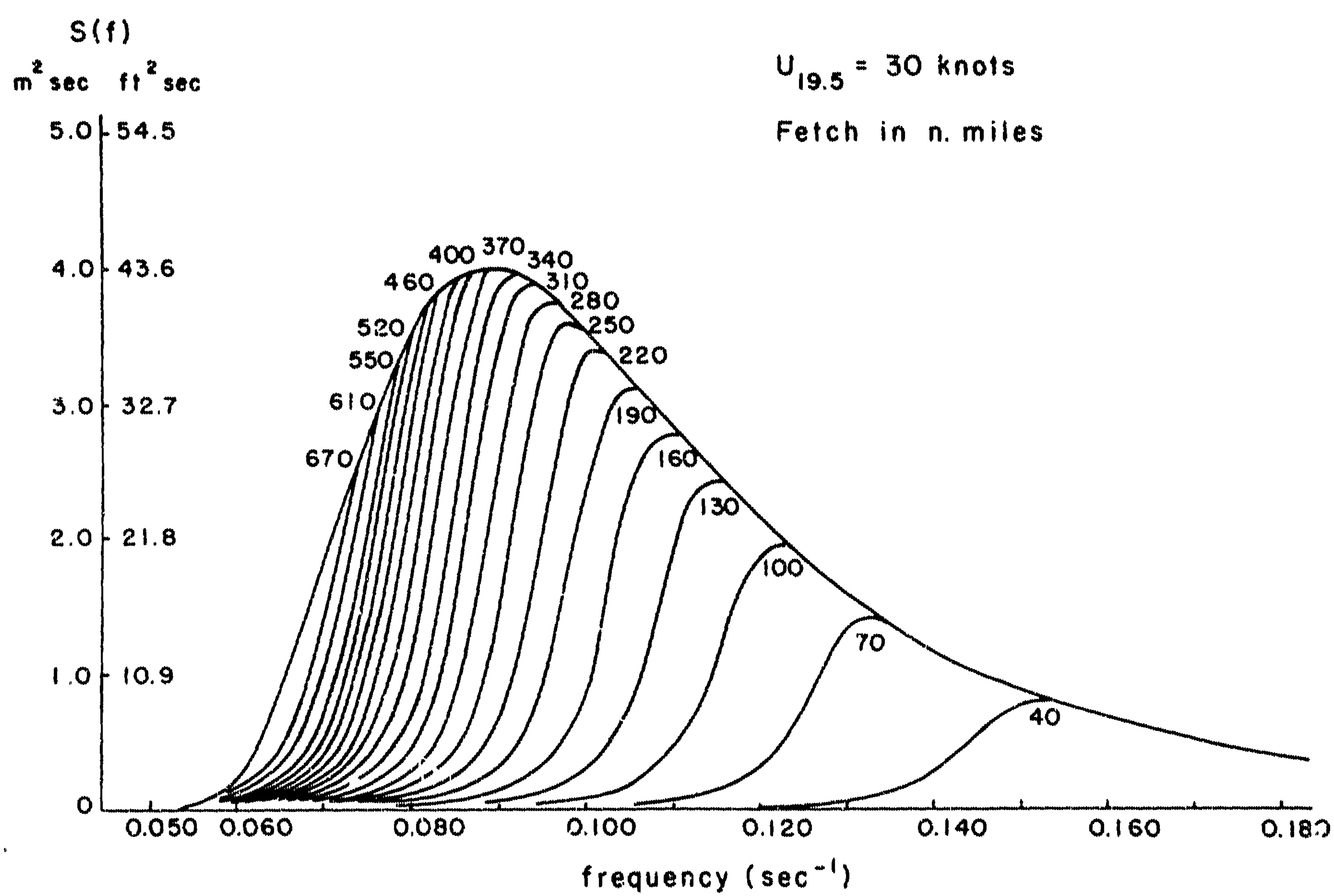

Fig. 4(c) Spectral growth witn respect to fetch.

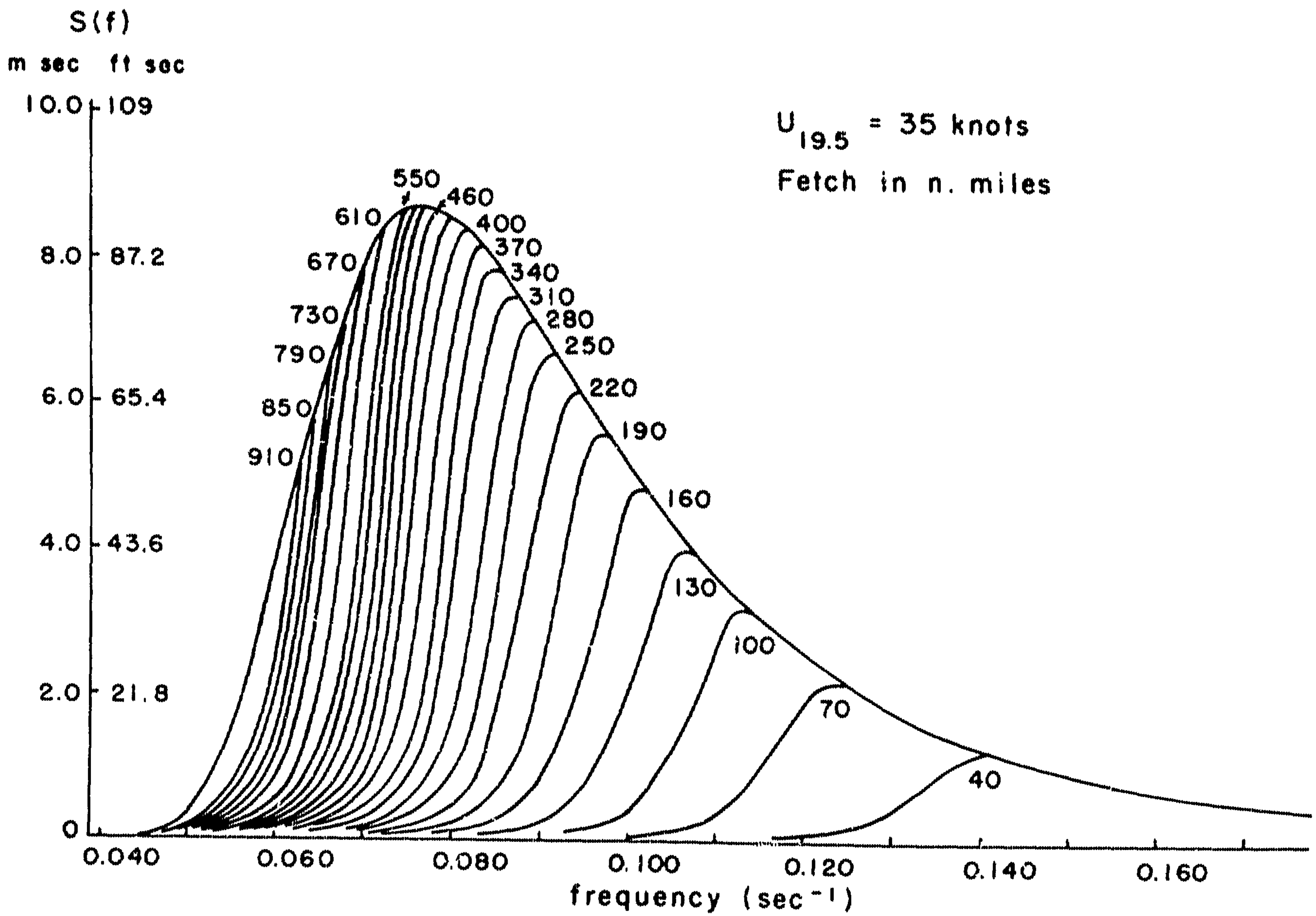

Fig. 4(d) Spectral growth with respect to fetch.

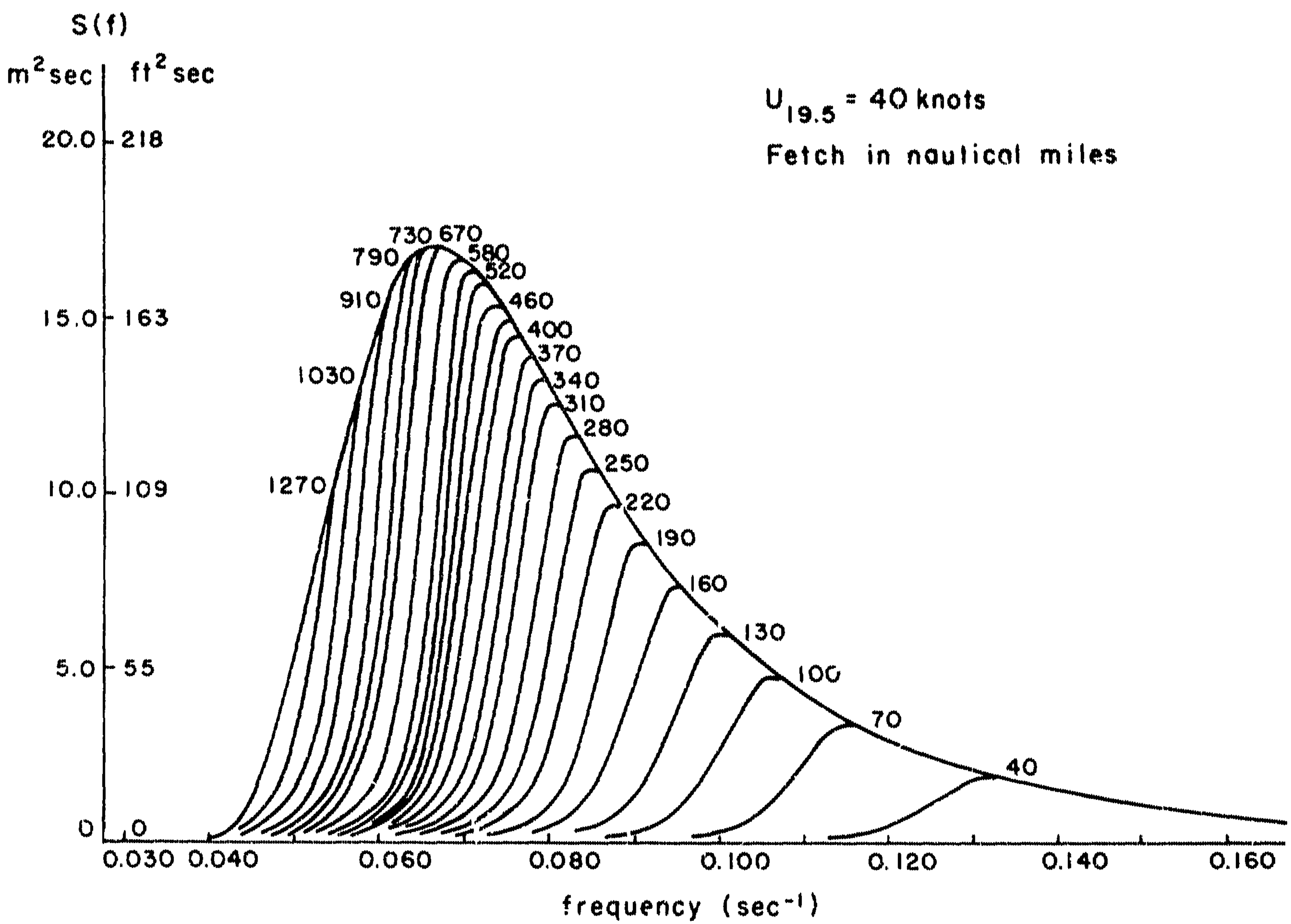

Fig. 4(e) Spectral growth with respect to fetch.

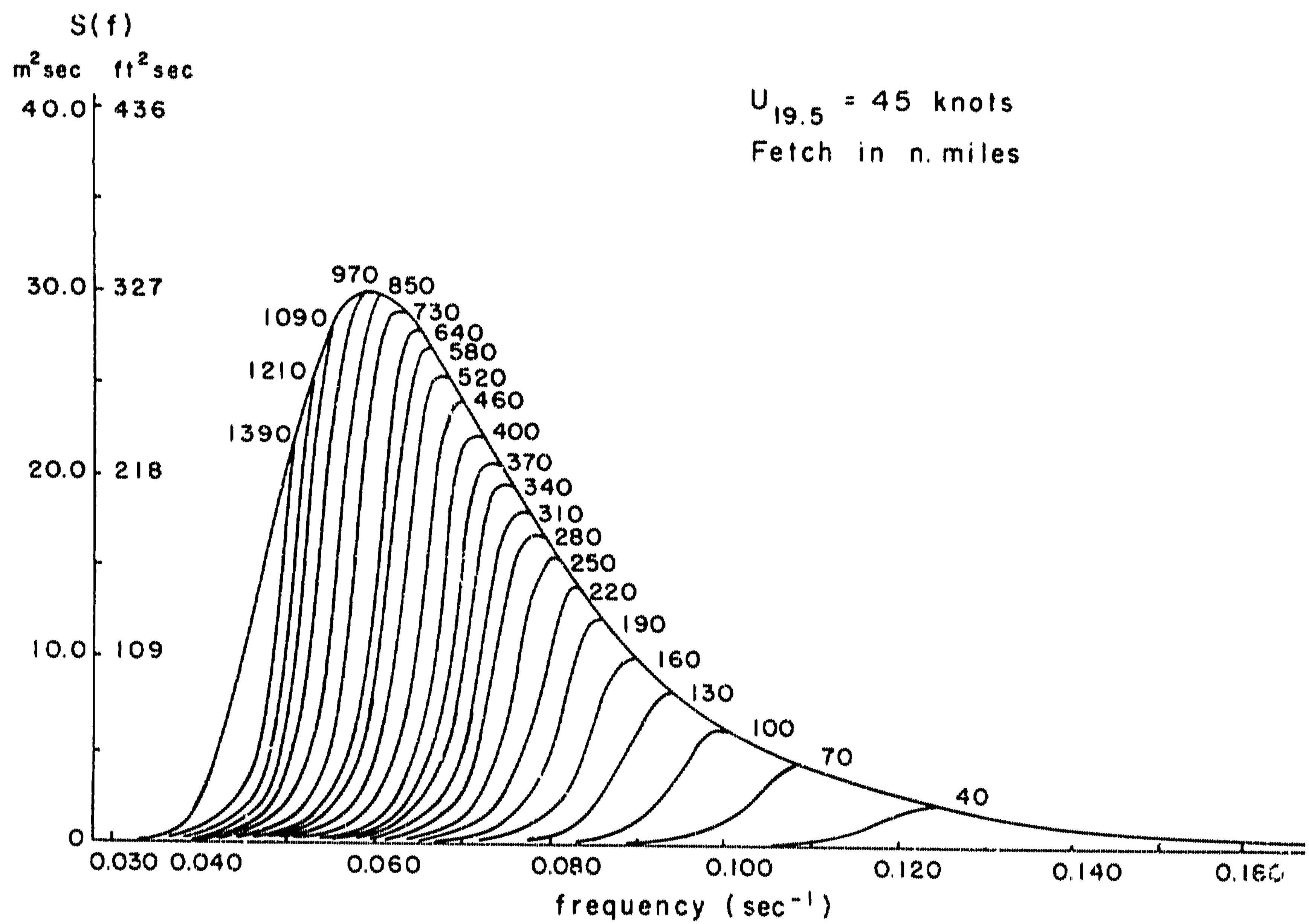

Fig. 4(*f*) Spectral growth with respect to fetch.

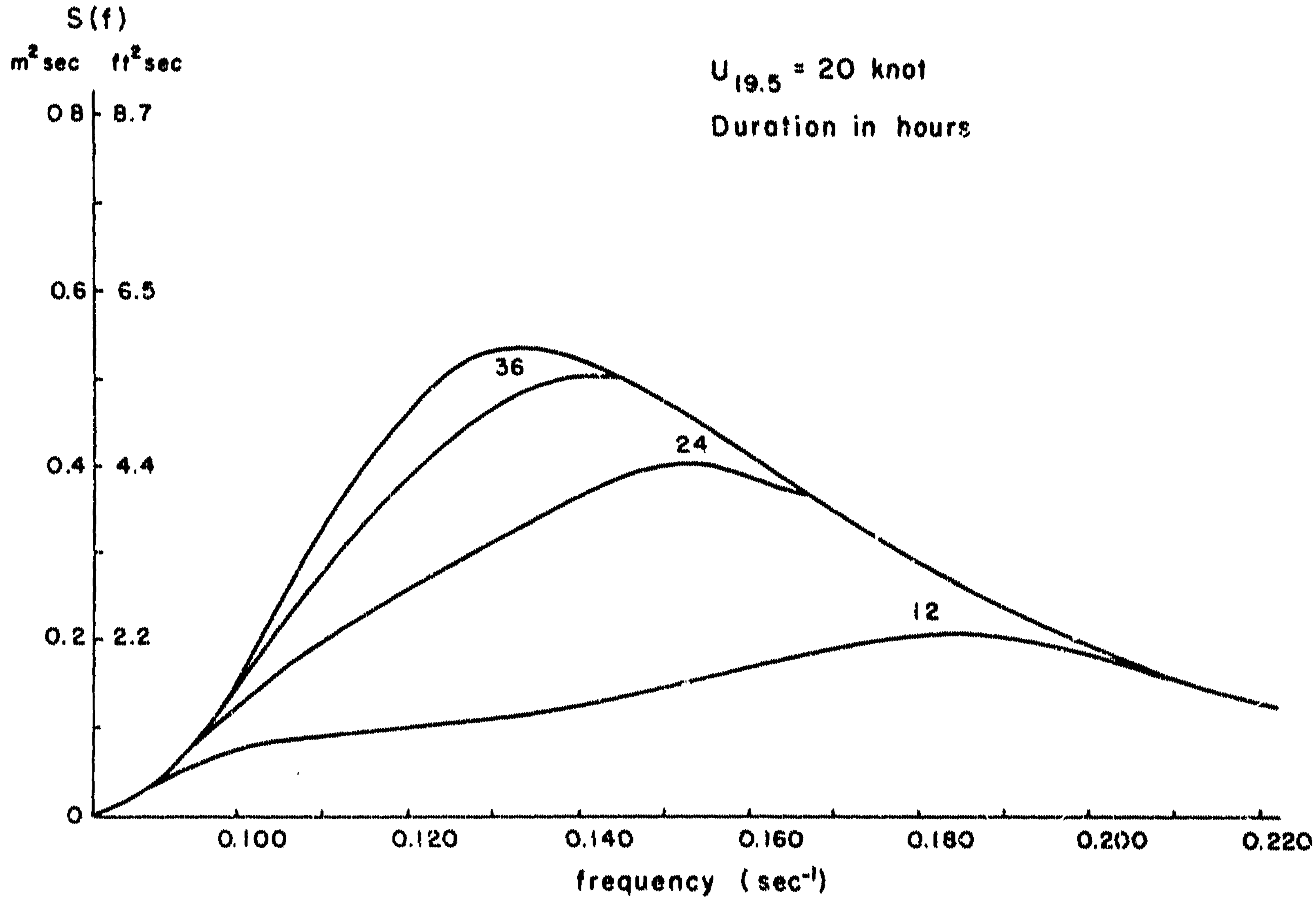

Fig. 5(a) Spectral growth in terms of original Miles and Phillips values.

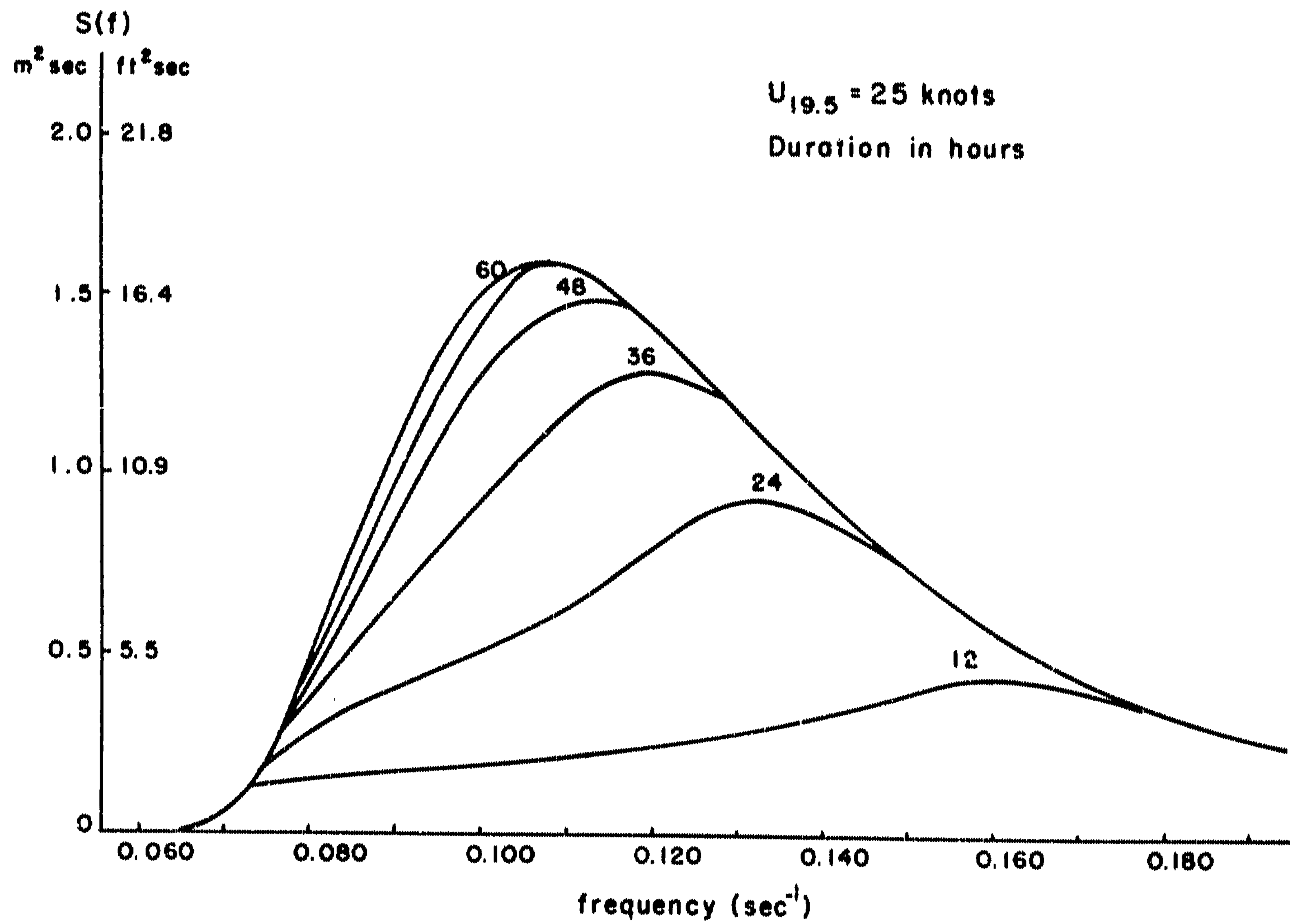

Fig. 5(b) Spectral growth in terms of original Miles and Phillips values.

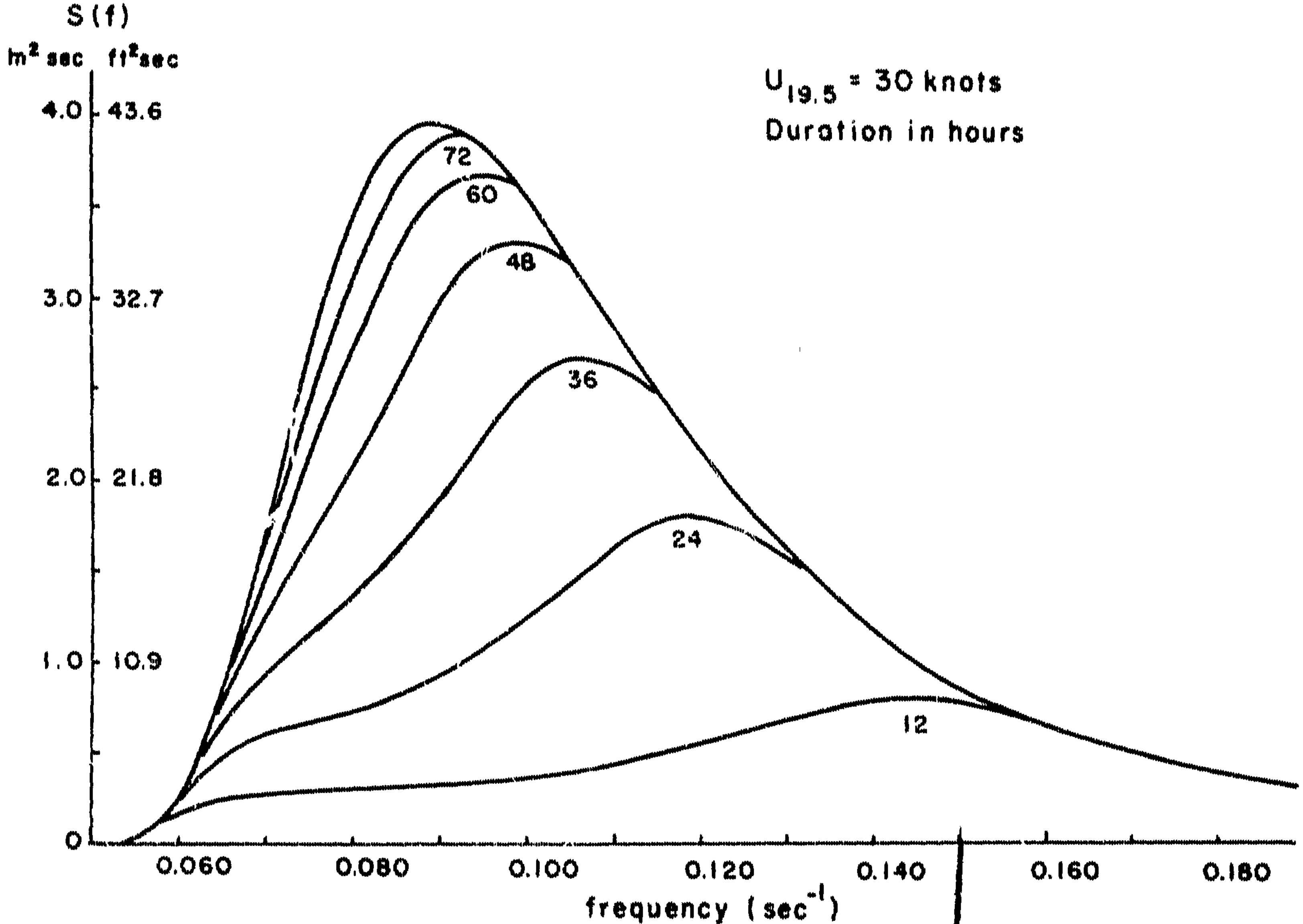

Fig. 5(c) Spectral growth in terms of original Miles and Phillips values.

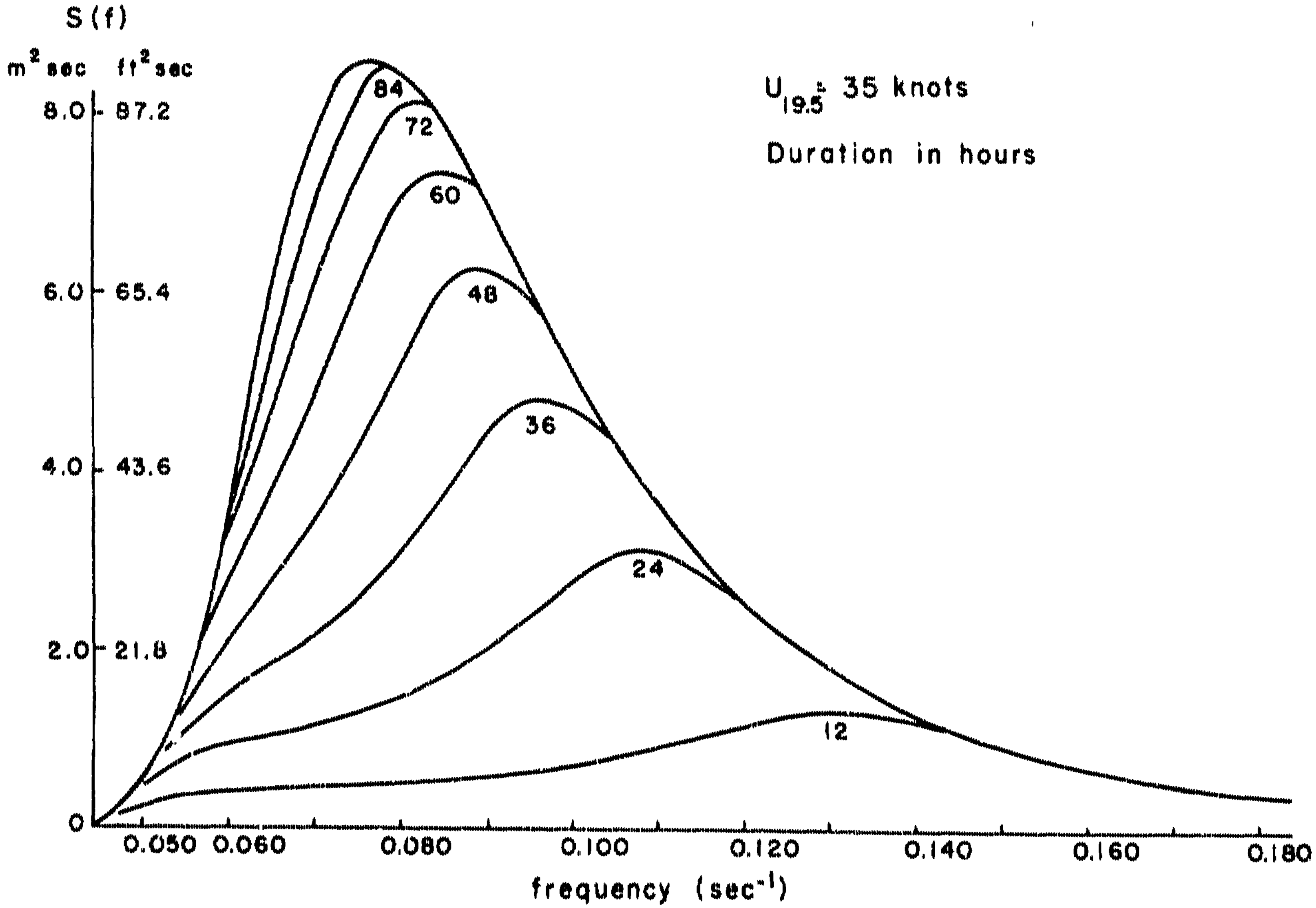

Fig. 5(d) Spectral growth in terms of original Miles and Phillips values.

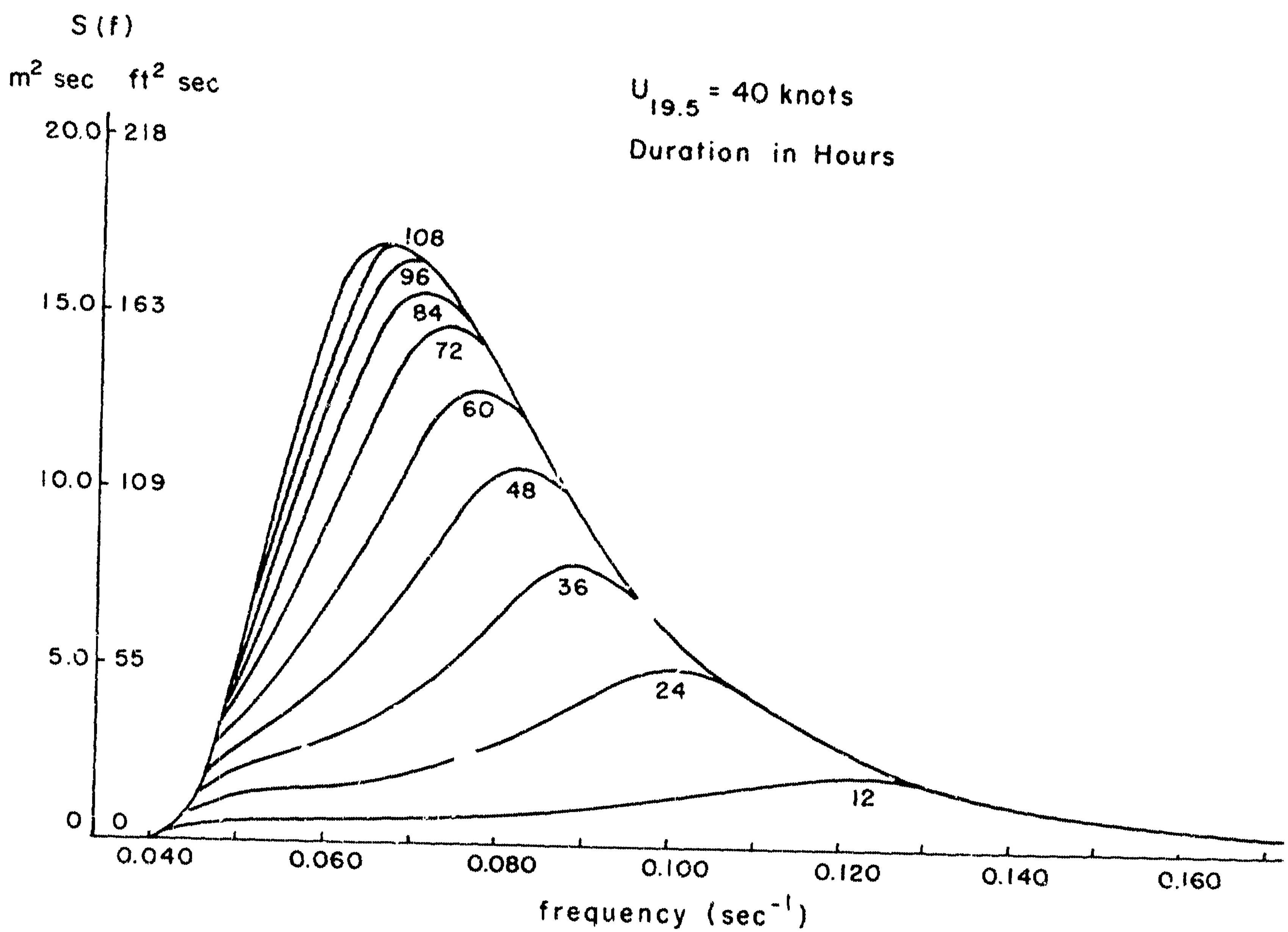

Fig. 5(e) Spectral growth in terms of original Miles and Phillips values.

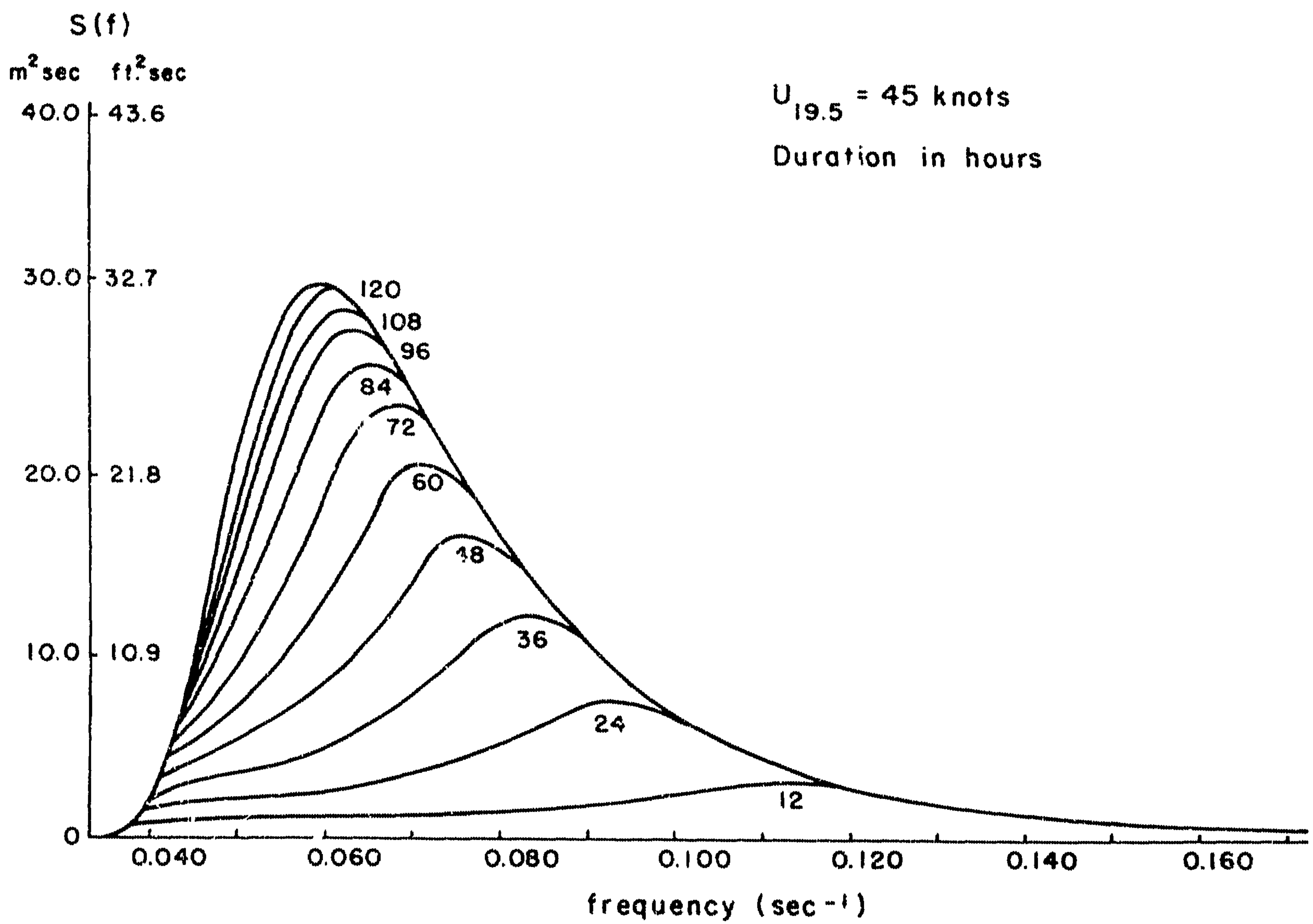

Fig. 5(f) Spectral growth in terms of original Miles and Phillips values.

shown. The low Miles term caused the slow growth rate in the high frequency range. These spectra are shown in Figures 5 (a) through (f).

8. Comparison with other results

The spectra obtained from the modified Miles-Phillips mechanism can now be compared with other proposed spectra.

Neumann (1953) assumed that the spectrum of a component starts to grow after the higher frequency components reach the fully developed spectrum. Therefore, the spectrum of the partially developed sea can be obtained from the spectrum of the fully developed sea by discarding the part of the curve below a certain frequency, and a sharp cut-off in the forward slope of the spectrum occurs. However, wave components which are lower than the cut-off frequency probably have a small amplitude, and the correction has been determined empirically. The wave spectra for a 40 knot wind at 19.5 m as given by Pierson, Neumann and James are shown in figure 6. The cut-off frequency and the correction frequency band are given, but a precise forward slope of the spectra is not given. Therefore, the spectra of figure 6 are approximated.

The spectra given in figure 6 are for a wind velocity of 36 knots of the PNJ method, but this is the same as the spectra for a 40 knot wind at 19.5 m. As Pierson (1964) showed, the spectral forms proposed by Neumann (1953), and by Pierson and Moskowitz (1964) are very close to each other for a 40 knot wind velocity.

In general, a comparison of figures 2(e) and 6 shows that the spectra of this paper grow faster than the PNJ spectra in the

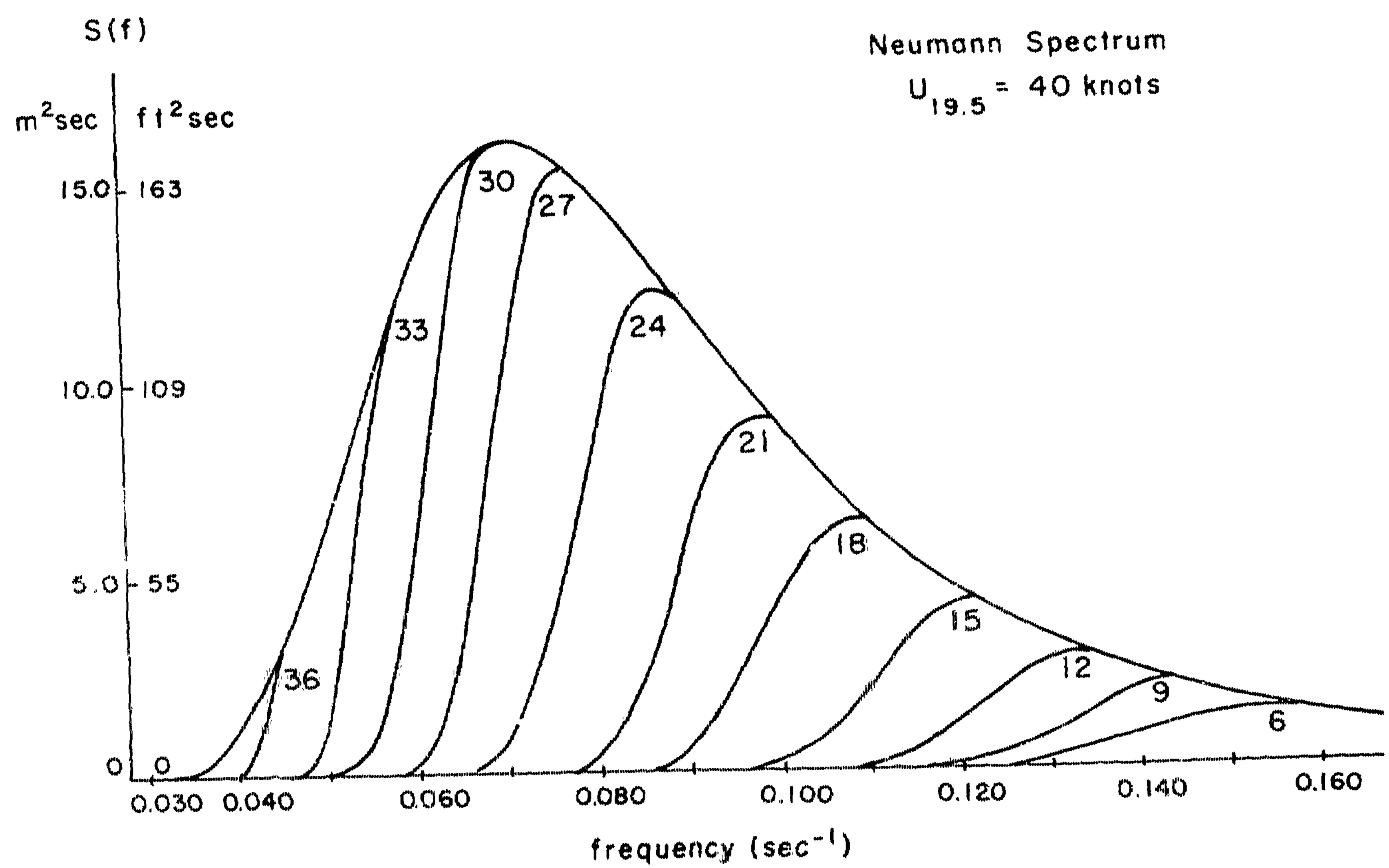

Fig. 6. Neumann spectrum for a 40 knot wind at 19.5 m.

early stages, and the growth rate decreases gradually with time. In contrast, the PNJ spectra grow relatively slowly at first, then they grow quite rapidly. In the spectral sense, the forward slope of the spectrum progresses steadily toward low frequenc.es.

Both have almost the same spectra at a duration of 30 hours. The PNJ spectra reach a fully developed spectra after 36 hours, but it takes more time for the spectra presented in this paper to fill in the fully developed curve. For practical purposes, the needed time to complete development after reaching 95% of the significant wave height can be neglected. The required duration for the fully developed sea is almost the same for the PNJ method and for the spectral growth of this study for a 40 knot wind.

Several spectral growths have been proposed so far, and discussed by Walden in Ocean Wave Spectra (1964). There are many discrepancies between these proposals. The shape of the spectra, the frequencies of the maximum energy, and the highest spectral densities are all different in each of the proposed spectral forms. The reason for these disagreements can be attributed to the different methods of observation, measurement, and analysis of the data. One of the interesting spectra was proposed by Bretschneider (1959). These spectra have the property that the energy in the higher-frequency region decrease with time. Another striking spectral form was proposed by Gelci, Cazalé and Vassal (1957). These spectra contain a fast-growing portion in the higher frequency region , which increases very slowly later, and another portion in the lower frequency region, which develops rapidly with

time. The latter part has a maximum energy after four hours for a 20 knot wind and the frequency of that maximum energy is not changed even though the spectra are growing. This is uncommon compared to the other kinds of growth that have been proposed.

All the different proposed spectra have different characteristics, so it is hard to say which are closest to the spectra presented in this study. There is still another way to check the growth rate. It is by a comparison of the significant wave heights in the many proposed growth curves.

Figures (7a) and (b) show the growth rate of significant wave height for duration and fetch for wind velocities from 20 through 45 knots, according to equations (8.2) and (8.4). These curves are similar to the ones which were obtained by Sverdrup and Munk (1947, as given in H. O., Misc. 11, 275). Their curves were plotted against "wave height", not significant wave height, but in the ordinary sense, both quantities can be considered to be the same. To avoid confusion, only 30 and 40 knot wind wave growth curves are plotted on this figure. In this study, wind velocity at 19.5 m is used, so the wind velocities were corrected for the difference in elevation between 10 and 19.5 meters.

Other duration and fetch curves for a 40 knot wind are shown in Figures 8(a) and 8(b). The curves of this study have a similar character to the curves of PNJ and Titov (1955) rather than the others in that it takes a longer time to reach a fully developed sea. The others show a faster growth.

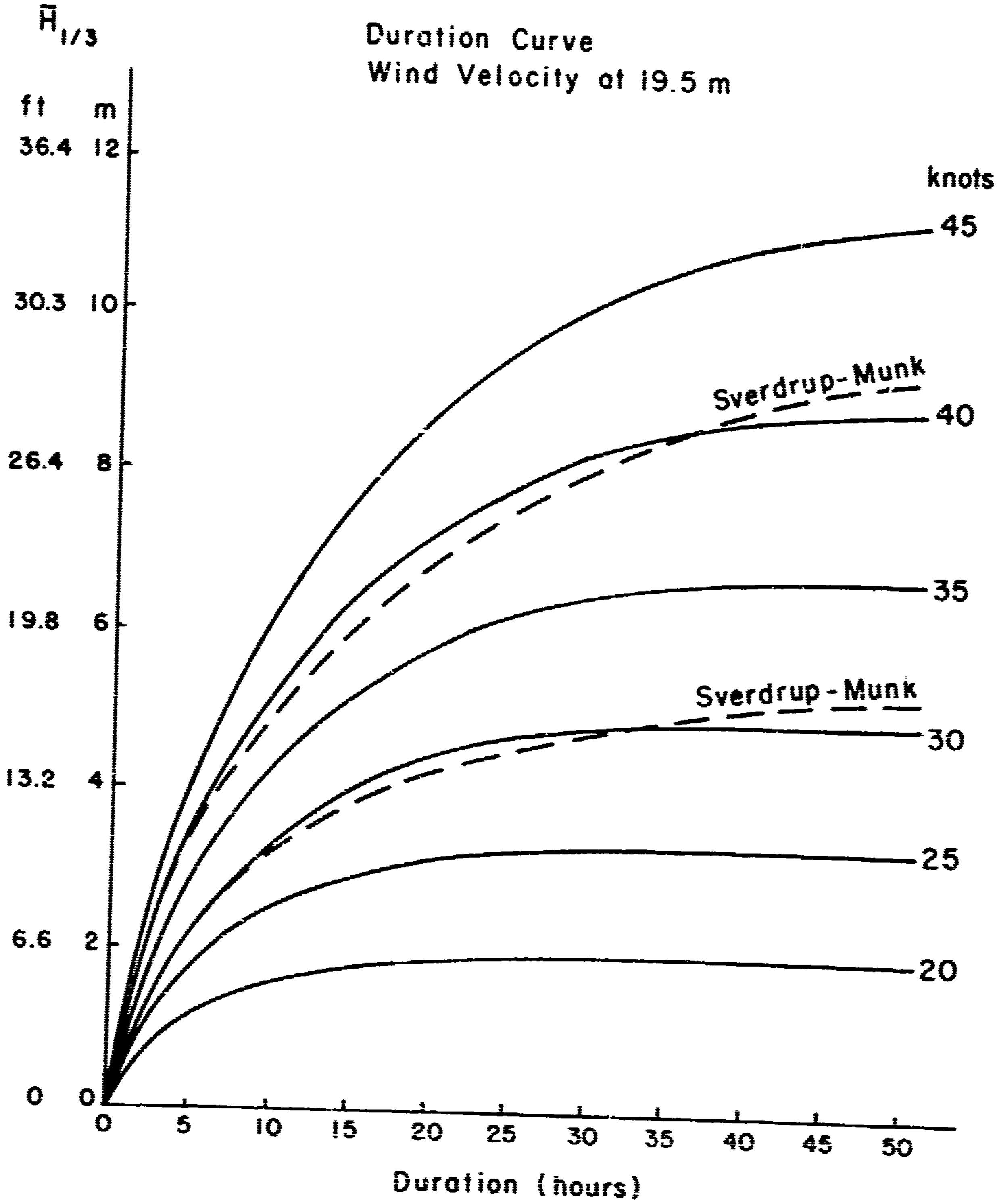

Fig. 7 (a). Growth of significant wave height with respect to duration.

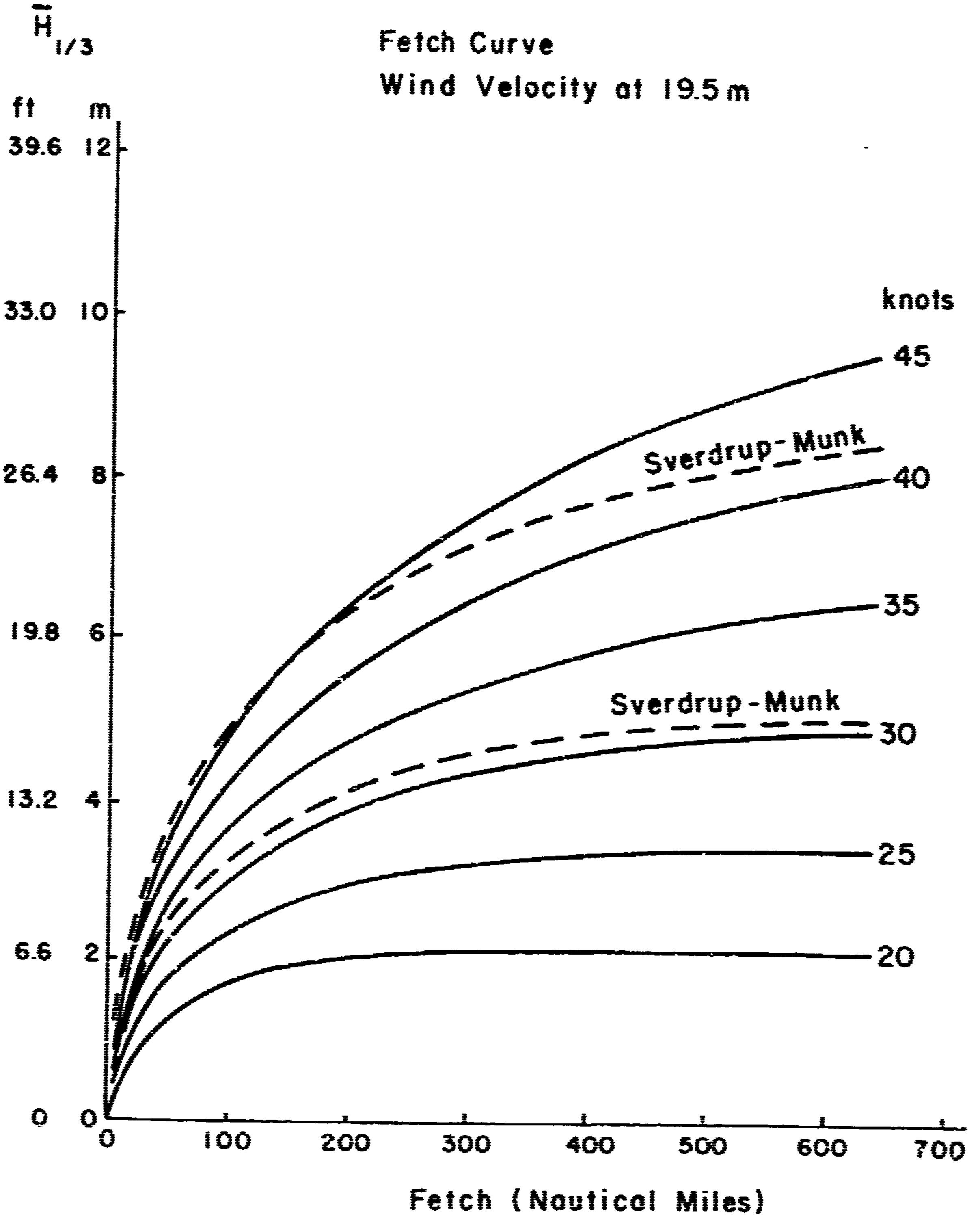

Fig. 7(b). Growth of significant wave height with respect to fetch.

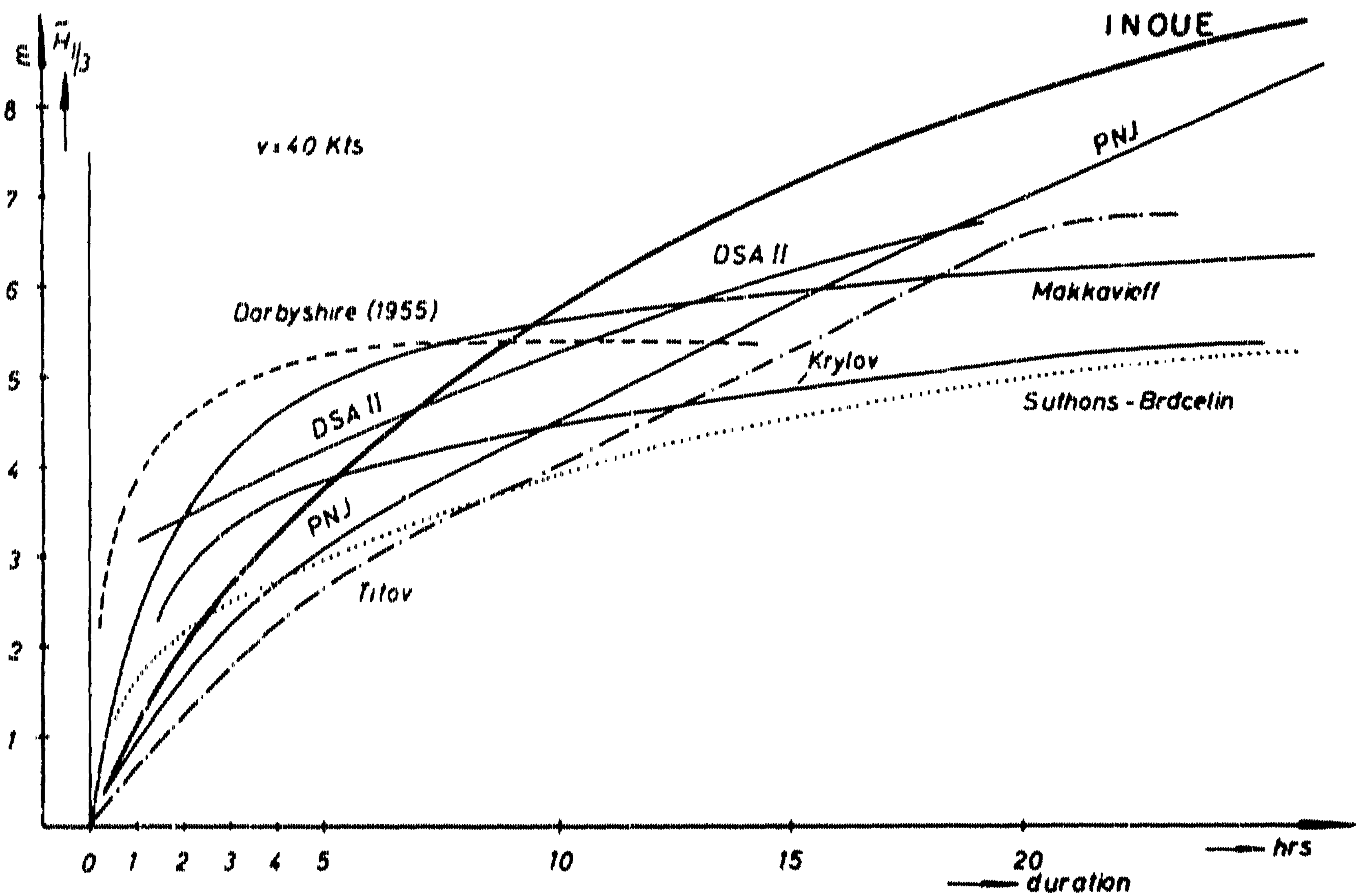

Fig. 8(a) Growth of significant wave height for a 40 knot wind at 19.5 m according to various theories.

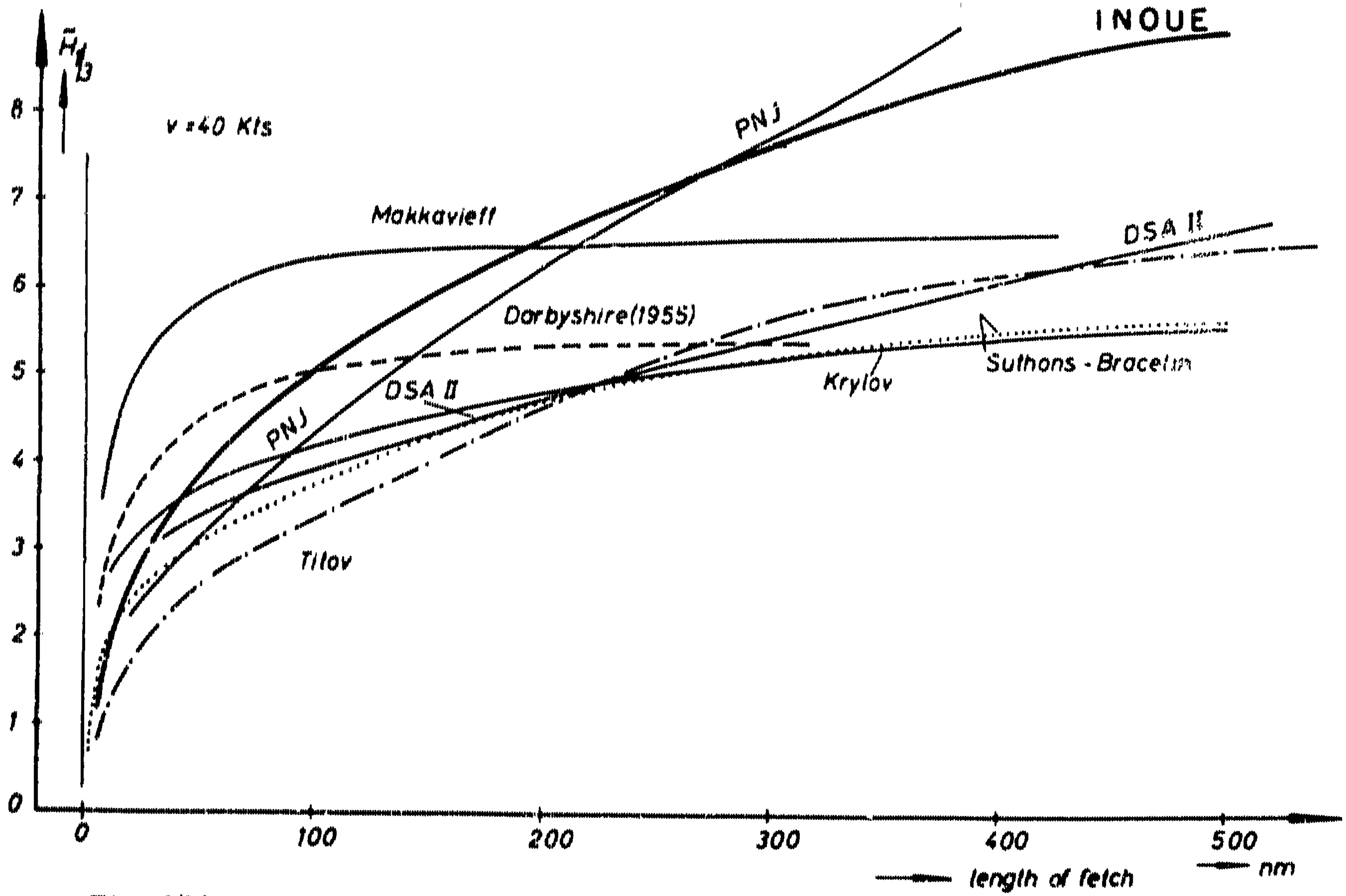

Fig. 8(b) Growth of significant wave height for a 40 knot wind at 19.5 m according to various theories.

This may be explained as an effect of the initial conditions. One of the most significant characteristics of the spectral growth of this study is whether or not the sea is at rest initially. If the sea is not at rest, there is some background spectrum present. Therefore, if a background spectrum exists, the Miles term, $B(\omega, u) \cdot S(\omega; t)$ affects the spectral growth strongly at all frequencies from the initial stage. Thus the growth is very different from the non-background spectral sea. When the sea is not at rest initially, the wave height will be observed to grow much faster.

A similar spectral growth computation was made by using equation (7.3) under the assumption of a background sea. The background level (assumed white) was approximated from the observed wave spectra for 20, 30, and 40 knot winds listed below:

wind velocity	variance (ft^2; frequency band of 1/180 sec^{-1})
20	0.011
30	0.100
40	0.280

The results of the computation are shown in figures 9 (a), (b), and (c). Clearly from these figures, the growth rate is very large. For instance, the spectrum for a 40 knot wind reaches the fully developed spectrum after 18 hours if there exists a background sea as given above. This is an important characteristic of the growth rates of this study.

In the open sea, background spectra are always expected to exist, and this leads to faster wave height growth. This may be

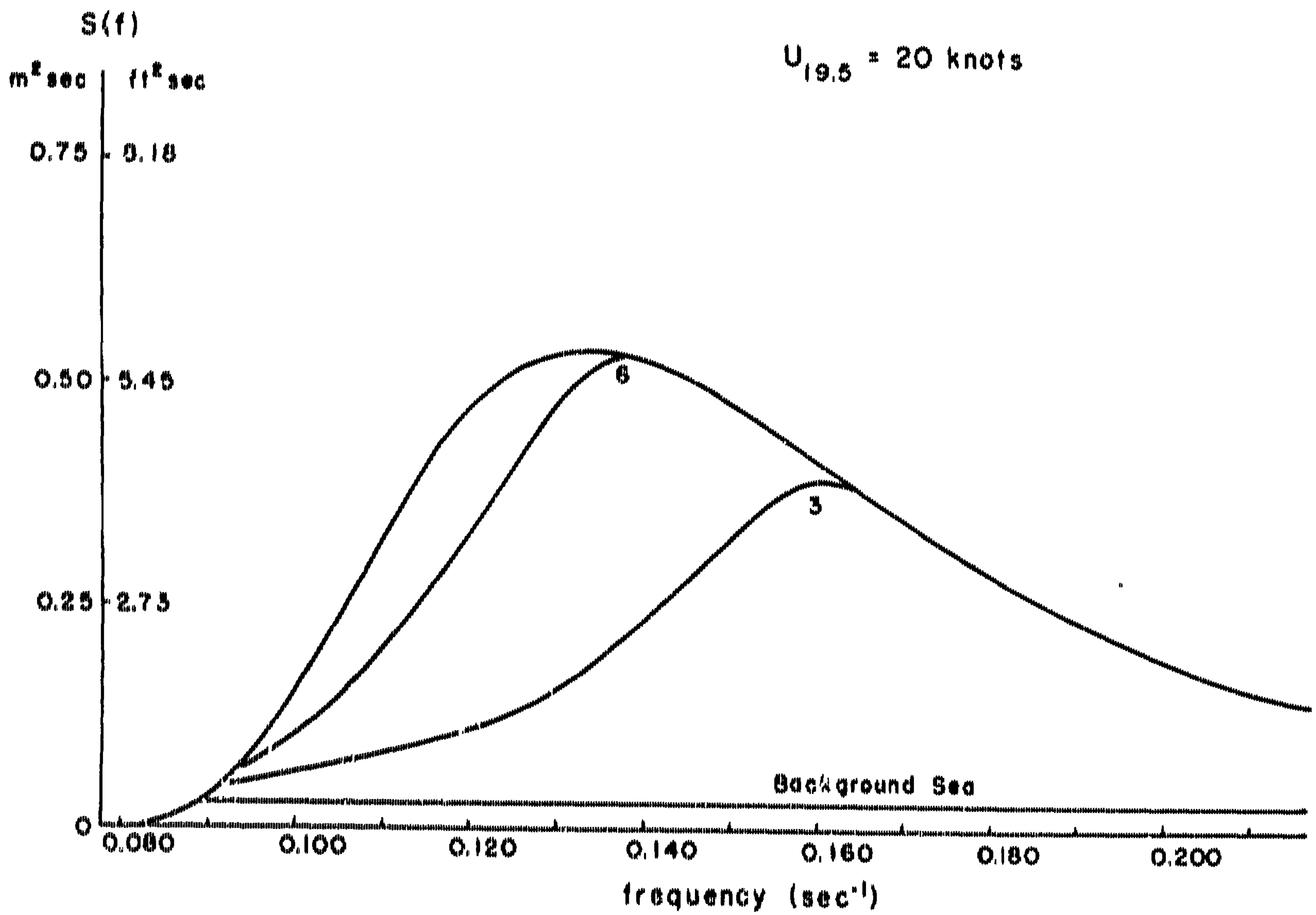

Fig. 9(a) Spectral growth from background sea.

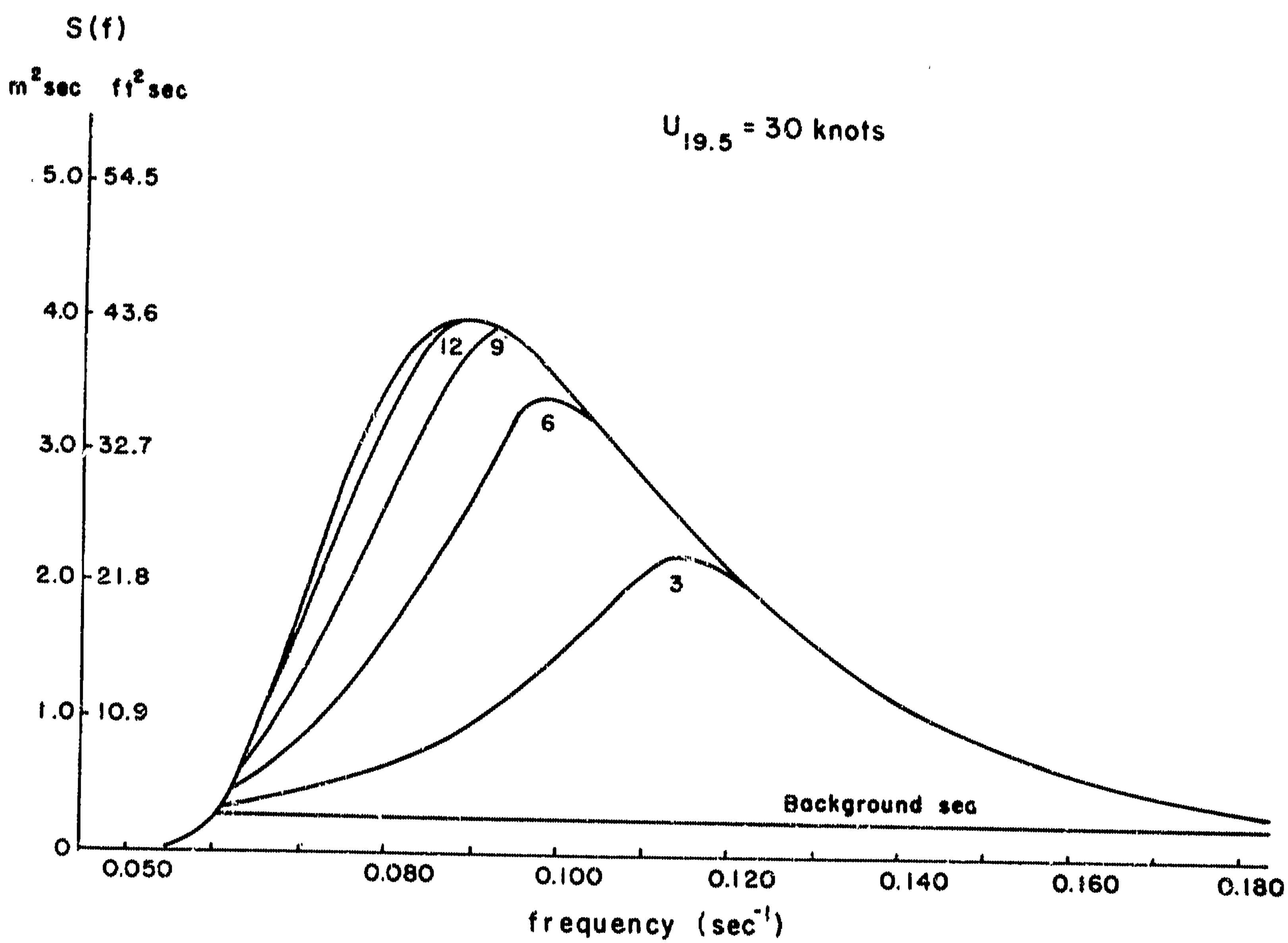

Fig. 9(b) Spectral growth from background sea.

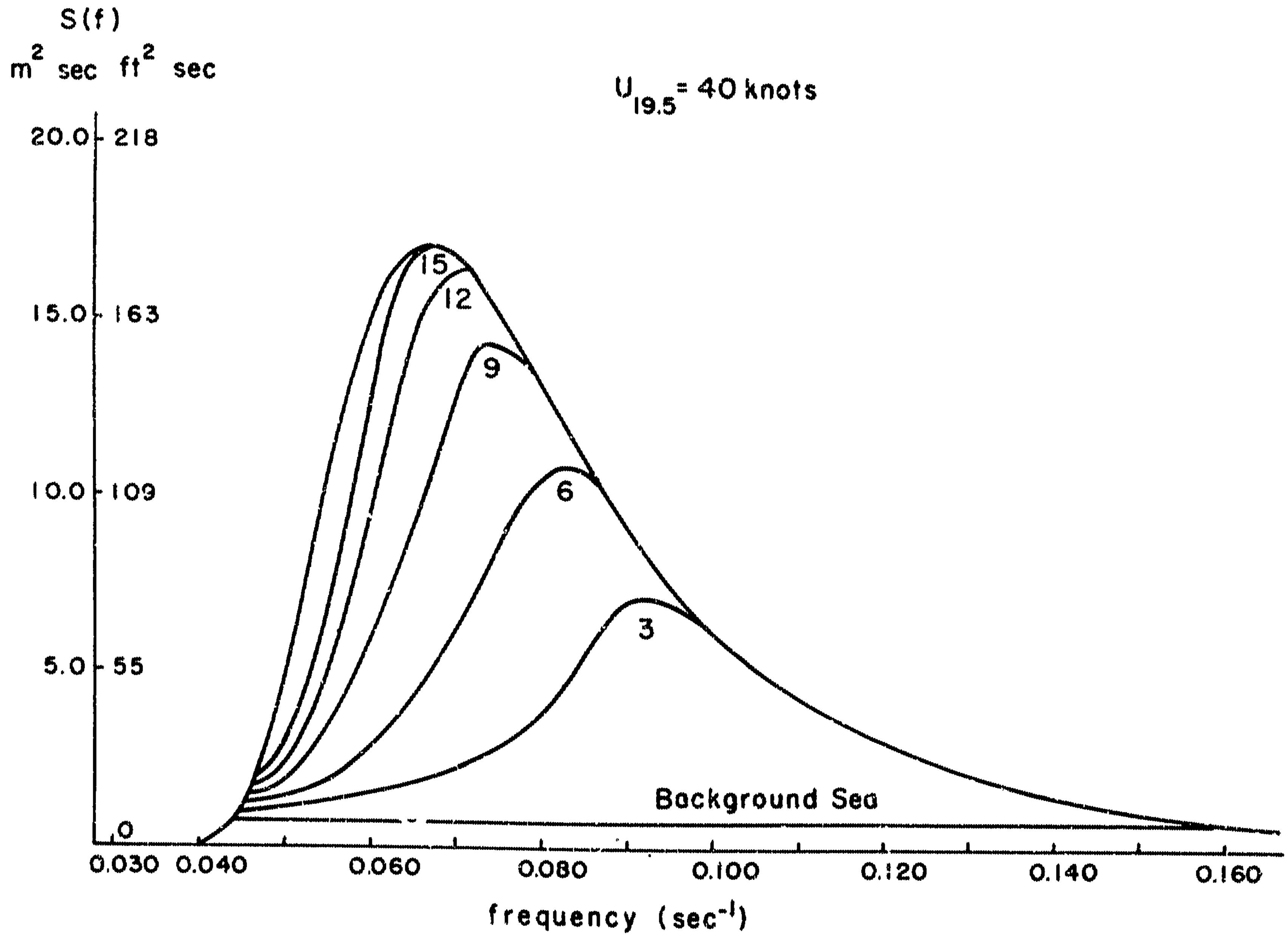

Fig. 9(c) Spectral growth from background sea.

one of the reasons for the discrepancies between the relatively slow-growth rate group and the rapid-growth group. The comments by Phillips (Ocean Wave Spectra, p. 37) are well substantiated by these results, and the correct description of low background spectra will be an important part of any numerical wave forecasting procedure.

9. Test run for observed spectra

Several spectra for the actual sea were computed and are shown in figures 10 (a) through (d). The observed spectra were obtained from wave records three or six hours apart. The wind velocities are scattered from about 40 knots to 60 knots. Unfortunately, a moderate wind changes its force and direction frequently and does not give a steady meteorological condition. This is the reason why the higher velocity winds were used.

All of the seas whose spectra were used in this study were accompanied by a strong low pressure in the North Atlantic. For example, the spectra shown in figure 10(a) were taken at Ocean Weather Station I at 00Z and 06Z on 8 Nov. 1959, and the low deepened to 948 mb at 06Z to the northwest of the station. All other spectra had almost similar conditions. The best fit for a sample hindcasted spectra is shown in figure 10(b). These spectra were taken at 00Z and 03Z on 17 Dec. 1959 at station J. The wind condition before that observation was as follows:

16 Dec.	15Z	210°	31 knots
	18Z	250°	32 knots
	21Z	250°	44 knots

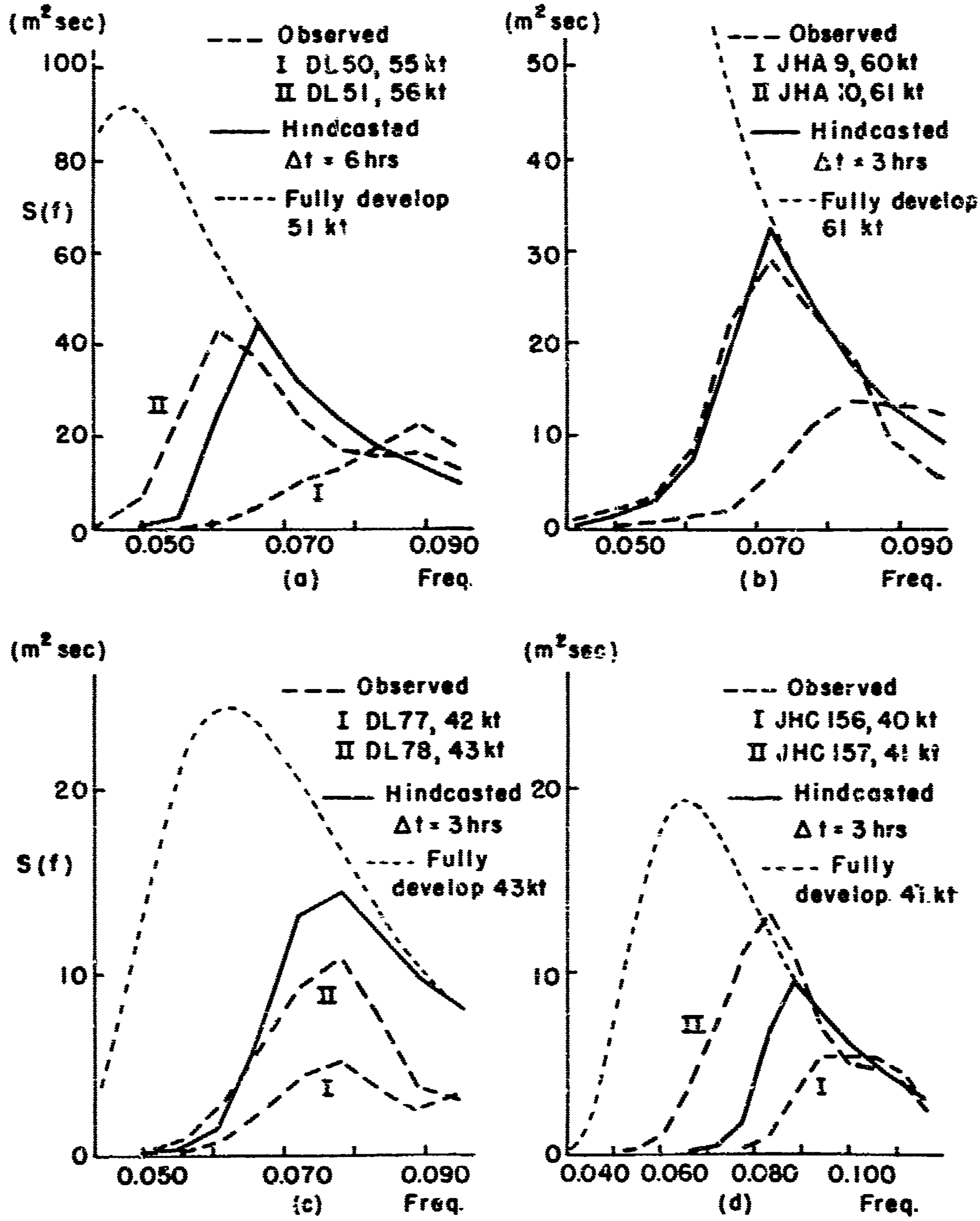

Fig. 10. Test calculations for actual observed spectra.

Then the wind started to blow very strongly,

	17 Dec.	00Z	250°	60 knots
		03Z	250°	61 knots

This storm weather situation and the wave spectra were presented in detail by Bretschneider et al (1962). The spectra are far from the fully developed spectrum, and less advective and nonlinear effects are possible in comparison with other spectra for moderate weather conditions. The computed spectrum can be assumed to be quite ideal and fits the observed spectrum quite well. Other observed spectra may have advective phenomena.

Furthermore, the observed spectral densities have sampling variability. The influence of sampling variability in the hindcasted spectra must also be considered. In figure 11, two examples are shown for a frequency of 0.061 for the spectra shown in figures 10 (a) and (c). The rectangle is bounded by the upper and lower 95% and 5% confidence intervals and by the ± 1 knot of the observed wind velocity. The bigger the overlapped area of the observed and hindcasted rectangles, the higher the reliability of the hindcasted spectra. Even though the spectra of figures 10 (a) and (c) show disagreement at a frequency of 0.061, the hindcasted spectra can be said to be in good agreement within the expected sampling variability.

10. Energy balance

The spectral growths of this study show reasonable rates even though the approach to the problem has been different from

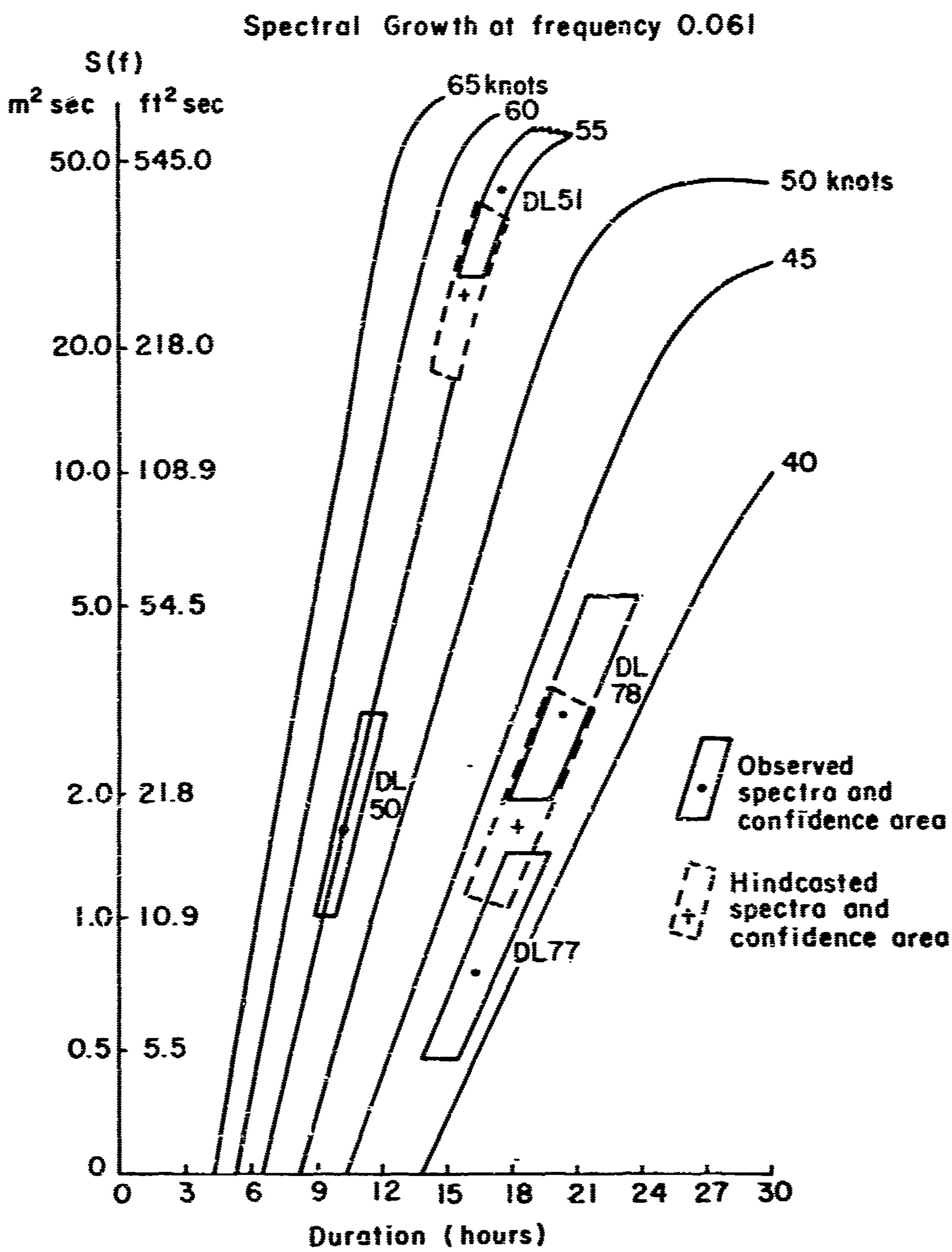

Fig. 11. Sampling variability of test calculations at frequency 0.061 .

the approaches described in Ocean Wave Spectra. However, one defect is lack of data. The data used were very limited and data from the different sources were combined. This suggests that another method is required to check this growth.

Energy considerations may be one solution to this problem. When the sea is fully developed, the energy input from the winds must be balanced by the energy dissipation in the water. If the dissipation assumption in equation (6.1) is neglected, the spectrum can keep growing higher than the saturated value.

Energy input estimations from the winds to the waves are very vague. One possible way for an energy investigation is the consideration of the amount of available energy in the atmosphere. Until the sea is saturated, the energy from the wind feeds the waves. After saturation, however, the energy input from the wind balances the dissipative energy in the sea. At a given height, the air moving with some mean horizontal velocity receives energy from the upper layer, and that energy equals

$$E_L = \tau u \tag{10.1}$$

where τ is the stress which is exerted on the air at that level, and u is the mean horizontal wind velocity. This stress is computed by using a drag coefficient when the vertical wind profile is assumed to be the logarithmic wind profile,

$$\tau = u^2 C_D \tag{10.2}$$

where C_D is the drag coefficient.

In this study, the drag coefficient obtained by Sheppard (1958) was used.

$$C_{10} = (0.80 + 0.114\, U_{10}) \times 10^{-3} \tag{10.3}$$

where U_{10} is the wind speed in meters per second at 10 m.

Until the sea reaches the saturated state, a part of the energy is transmitted to the sea surface, and raises the wave height. After saturation, the energy transmitted from the air should be balanced by the energy dissipated in the breaking waves. The transmitted energy at a given level in the air can be more than the energy dissipated in the water because it can be dissipated in the lower layers of the air. If the dissipated energy is much larger than the transmitted energy, the spectral growth of this study must be rejected. The transmitted energy is computed at the level of one half of the significant wave height.

Calculations of the energy transmitted downward at the level of one half of the significant wave height and the energy dissipated in the breaking waves of a fully developed sea are shown in figure 12. The two curves are close. The energy input from the winds is not known to good accuracy, but this energy consideration can be considered one of the means to check the growth rates of this paper.

11. Conclusion

The spectra were computed for the wind velocity from 20 to 45 knots and are shown in the figures. The growth rate of this study is not rapid, but it is similar to the growth rate of Pierson,

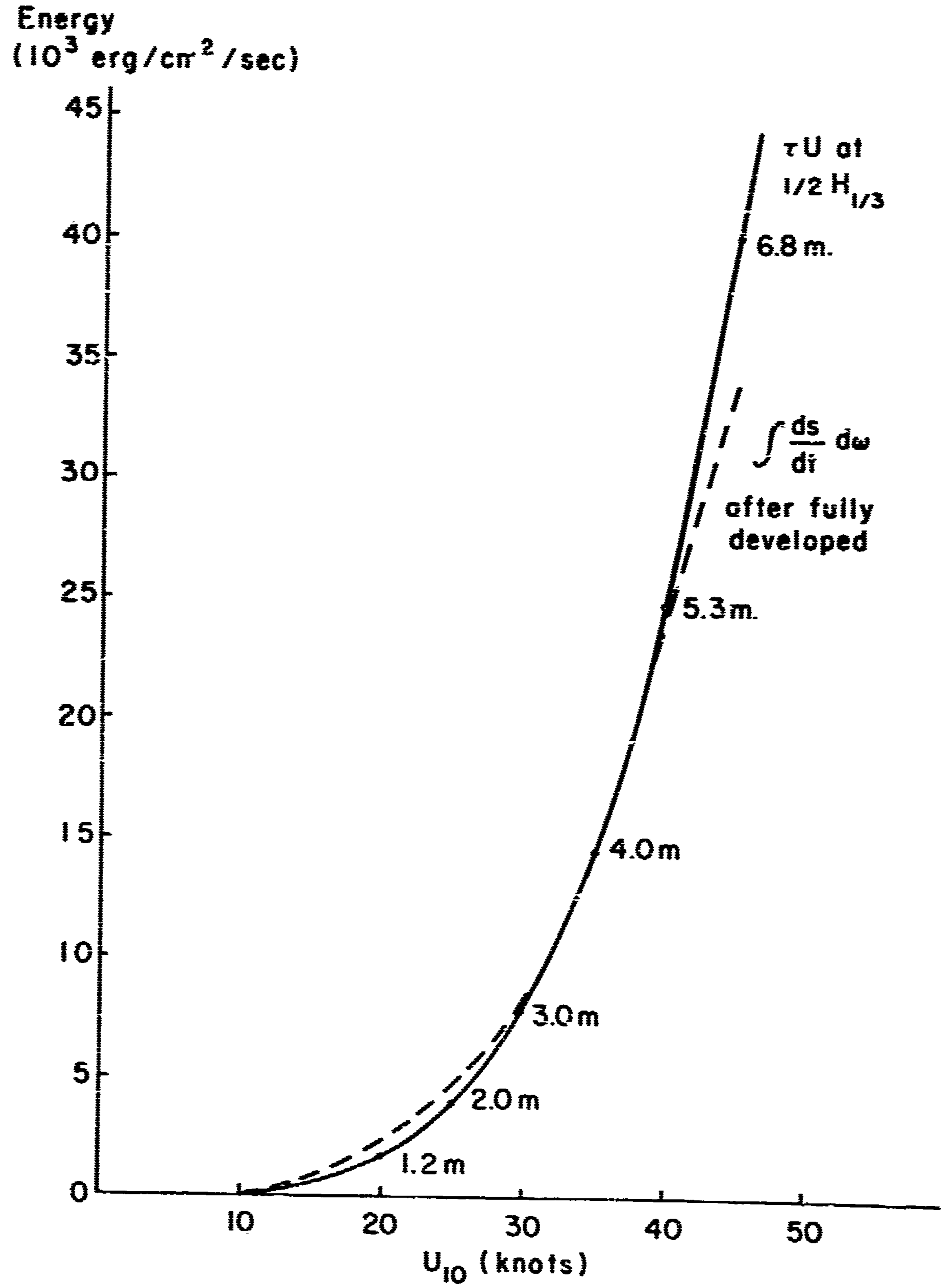

Fig. 12. Energy transported downward in atmosphere and dissipated energy in the waves.

Neumann and James (1955), and of Titov (1955) in a general sense. The spectrum of this study needs 30 hours or 600 nautical miles to reach the 90% of the fully developed significant wave height for a 40 knot wind at 19.5 m. These figures seem large in comparison with the other proposed values like the growth rate of Darbyshire. But these discrepancies may be explained by the existence of background spectra. Also the Phillips term affects the growth rate quite strongly.

The lack of more adequate data was the weak point of this study, so an energy balance between the atmosphere and the waves was considered, and the result showed good agreement. Also test calculations were made to check the actual observed spectra, and some of them showed good reliability.

The result of this study can be used for a practical wave forecasting technique instead of the empirical growth relation. But even a little change of the constants obtained in this study affects the growth rate of spectra. Further study in terms of a numerical hindcasting procedure should improve on these results.

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APPENDIX

List of Code Symbols

Symbol	Record Number	Wind velocity (knots)	Duration (hours)	Position
△	JH 26 - JH 27	50-50	3	A
O	JHA 9 - JHA 10	60-61	3	J
*	JHC 38 - JHC 39	45-42	6	J
x	JHC 73 - JHC 74	43-43	3	J
•	JHC 74 - JHC 75	43-44	3	J
I	JHC 156 - JHC 157	40-41	3	J
▽	DL 50 - DL 51	55-56	6	A